# Steady state heat transport in magnetized plasmas with magnetic islands and local stochastic fields

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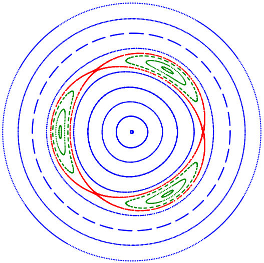
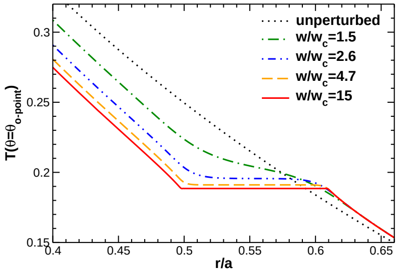
# Motivation

# Motivation: Magnetic islands

- magnetic reconnection (example: $3/2$-island)
- field lines wander around island surfaces
- parallel transport ($\chi_{||}$): fast, long distance
- perpendicular transport ($\chi_\perp$): slow, short distance
- parallel + perp. transport $\Rightarrow$ temperature flattening
- scale island width $w_c$: parallel $\approx$ perpendicular transport



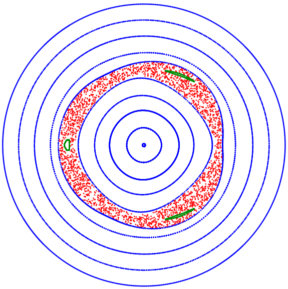


# Motivation: Neoclassical tearing mode

- bootstrap current: toroidal current driven by $\nabla_r p$
- perturbed by island temperature flattening
  - effective lack current in the island o-point region
  - acts as a driving term for further island growth
  - $\Rightarrow$ neoclassical tearing mode (NTM)
- NTM stability strongly depends on temp. distribution
- exact heat flux computations are important
- analytical theory is limited to the cases $w/w_c \rightarrow 0$ and $w/w_c \rightarrow \infty$, see Fitzpatrick (1995)
- AIM: numerical computations for realistic parameters

# Motivation: Stochastic layers

- overlapping magnetic islands destroy flux surfaces

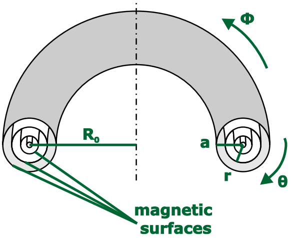


- field lines move through stochastic layer in a seemingly random way (increases radial heat transport)
- AIM: determine radial heat conductivity of highly stochastic layers and compare to analytical predictions

# Model

# Model: Geometry

- equilibrium: circular cross section, large aspect ratio
- q-profile with 0.9 in the center and 4.0 at the edge



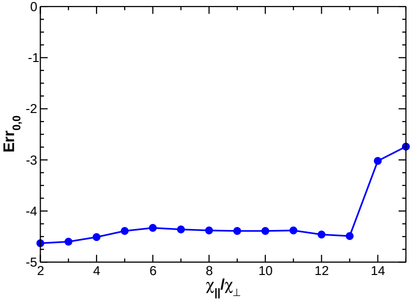

- coordinates: helical, unsheared, rational helicity $q_c$ (adapted to problem)
- grid points in radial and poloidal direction
- Fourier expansion in toroidal direction

# Model: Heat diffusion equation

- Solve steady state heat diffusion equation $\nabla \vec{q} = P$
  - $P$: power source (heating)
  - $\vec{q} = -n\chi_{||}\nabla_{||}T - n\chi_{\perp}\nabla_{\perp}T$: heat flux density
  - $n$: particle density
  - $\chi_{||}/\chi_{\perp}$ typically between $10^7$ and $10^{11}$
- common numerical schemes: error $\propto \chi_{||}/\chi_{\perp} \Rightarrow$ exact coordinate alignment
- virtually impossible for time-dependent problems
- new scheme developed by Günter et al. (2005)
  - conserves self-adjointness of parallel transport operator
  - temperature and heat flux grids shifted against each other
  - Fourier cut-off is performed at a certain heat flux order

# Model: New numerical scheme

- deviations from analytical solution for non-trivial test case
- small numerical errors
- independent from $\chi_{||}/\chi_{\perp}$ up to $10^{13}$

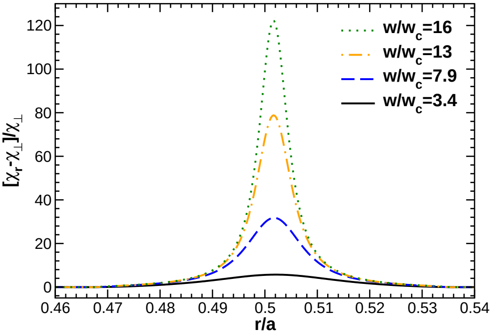



- code benchmarks were also performed

# Magnetic island results

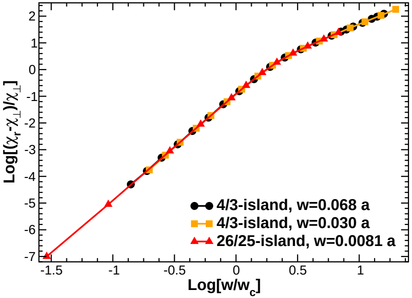
# Magnetic islands: Radial heat diffusivity $\chi_r$

- effective radial heat diffusivity $\chi_r$
  - increased in the island region
  - maximum at the resonant surface
- $4/3$-island with $w = 0.068a$:



# Magnetic islands: Scaling of $\chi_r$

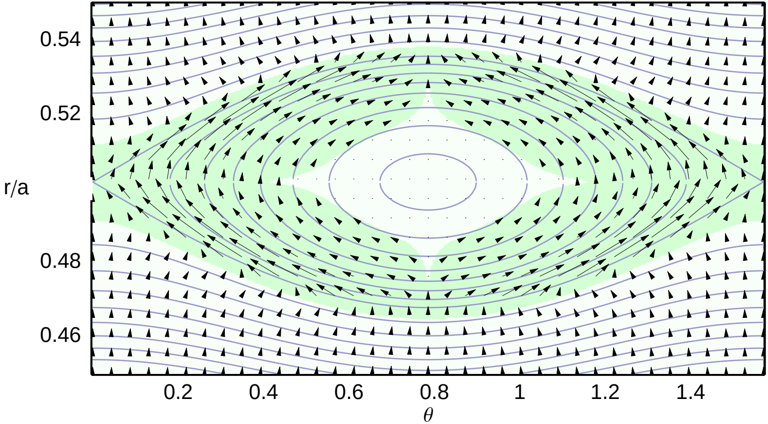
- $\chi_r$ at the island resonant surface




- two different regimes: $\chi_r \propto \left(\frac{w}{w_c}\right)^4$ resp. $\chi_r \propto \left(\frac{w}{w_c}\right)^2$
- depends on $w/w_c$ only

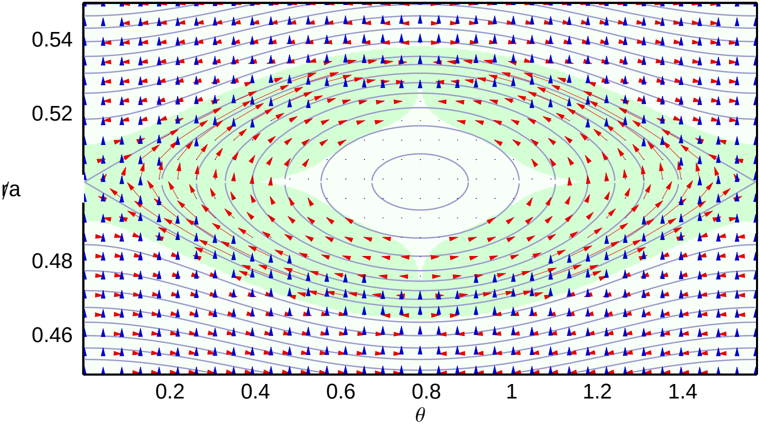
# Magnetic islands: Heat flux

- Total heat flux in the island region (4/3-island, $w = 0.068a$)
- $w/w_c = 3.4 \Rightarrow$ largely flattened island



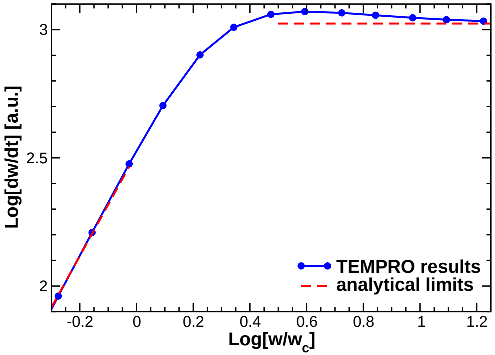
# Magnetic islands: Heat flux components

- Same case
- Parallel (red) and perpendicular (blue) heat flux



# NTM stability: Comparison to analytical limits

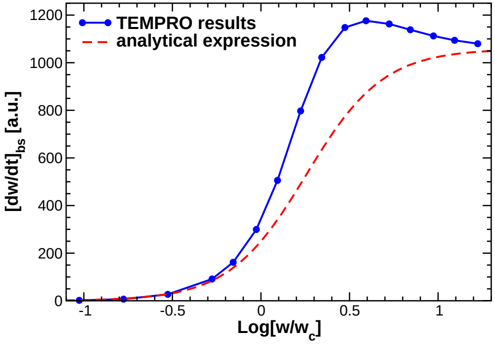
- NTMs destabilized by seed island temperature pert.
- Neoclassical contribution to island growth rate:




- Numerical results match analytical limits

# NTM stability: Comparison to analytical matching

- Fitzpatrick performed matching of the analytical limits

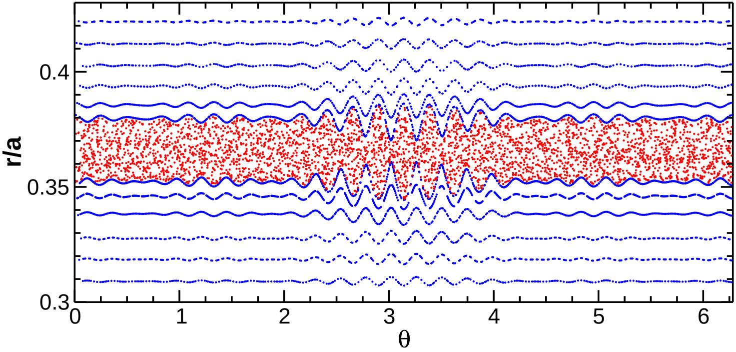



- underestimates island growth rate significantly
- can make the difference between stable and unstable

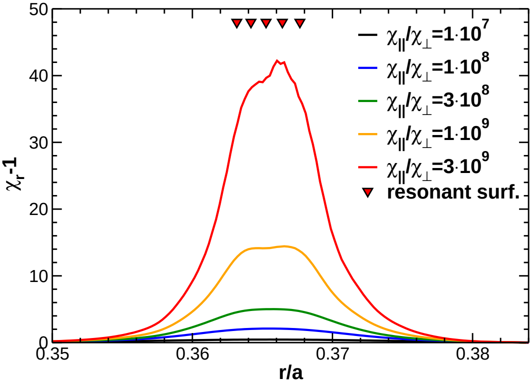
# Results for highly stochastic layers

# Stochastic layers: Flux surface destruction

- flux surfaces destroyed for $s = \frac{(w_1+w_2)/2}{|r_{res,1}-r_{res,2}|} \gtrsim 1$
- test case:
  - 5 islands: $24/23$, $25/24$, $26/25$, $27/26$, $28/27$
  - total stochasticity $s = 48.5 \gg 1$



# Stochastic layer: Radial heat diffusivity $\chi_r$



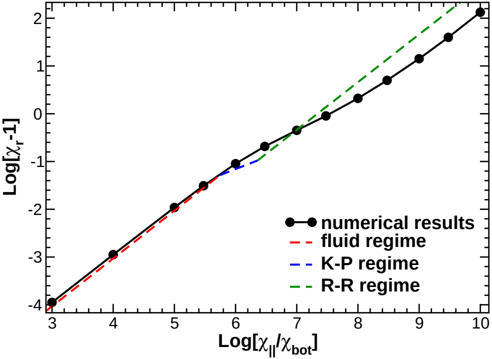

- radial heat diffusivity $\chi_r$ strongly increased in the region of the stochastic layer

# Stochastic layers: Analytical theories

- analytical theories for heat transport across highly stochastic layers ($s \gg 1$): review by Liewer (1985)
- Rechester-Rosenbluth regime
  - low collisionality
  - electrons basically follow the stochastic field lines
  - transport dominated by field line diffusion
- Kadomtsev-Pogutse regime
  - medium collisionality
  - increased importance of electron diffusion
- fluid regime
  - high collisionality
  - transport dominated by electron diffusion

# Stochastic layer: Scaling of $\chi_r$

- $\chi_r$ in the center of the ergodic layer:




- the three analytically predicted regimes can be observed
- ranges of validity do not coincide

# Summary

- implemented code for heat diffusion computations
- demonstrated computations with unaligned coordinates
- radial heat diffusivity $\chi_r$ for islands
- $[dw/dt]_{bs}$ for neoclassical tearing modes; widely used analytical matching underestimates island growth!
- $\chi_r$ at highly stochastic layers with realistic parameters
  - found the analytically predicted regimes
  - different ranges of validity
- experience from this work will be used to implement a nonlinear MHD code for plasma edge examinations

# Acknowledgements / References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Fitzpatrick, R. *Phys. Plasmas*, 2(3), 1995.

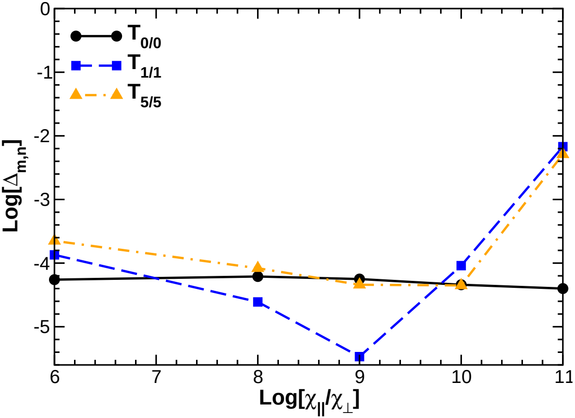
Günter, S., Yu, Q., Krüger, J., and Lackner, K. *Journal of Computational Physics*, 209 pp. 354, 2005.

Liewer, P. C. *Nuclear Fusion*, 25(5) pp. 543, 1985.

# References

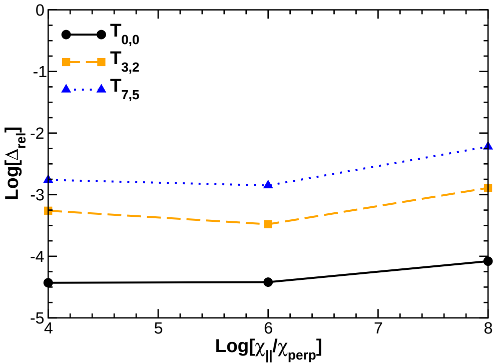
# Island code benchmark with TM1

- 1/1 magnetic perturbation with $w = 0.118a$
- at $\chi_{||}/\chi_{\perp} = 10^{11}$: $w/w_c = 13.8$



# Ergodic code benchmark with TM1

- $4/3$ and $3/2$ magnetic perturbations
- $s = 1.4$
- $w/w_c$ between 0.34 and 3.4
- Relative differences for representative modes:



# Analytical test case

$$\vec{B} = \nabla\Psi \times \hat{e}_\phi, \quad (1)$$

$$\Psi(r,\theta,\phi) = \Psi_0(r) + \Psi_1(r,\theta,\phi), \quad (2)$$

$$\Psi_0(r) = \Psi_0 \left(\frac{r}{a}\right)^2 \left(\frac{r-r_0}{a}\right)^2, \quad (3)$$

$$\Psi_1(r,\theta,\phi) = \Psi_1 \left(\frac{r}{a}\right)^2 \cos(m\theta - n\phi). \quad (4)$$

- analytical solution:

$$T(r,\theta,\phi) = \Psi(r,\theta,\phi), \quad (5)$$

# Analytical test case (continued)

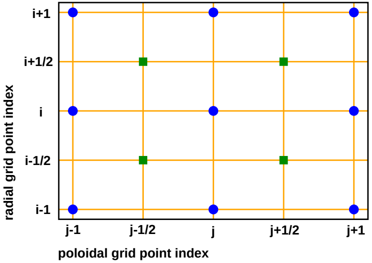
- boundary condition: $T(r = a) = \Psi(r = a)$
- analytical solution: parallel heat flux $\vec{q}_{||} = \hat{b}(\hat{b} \cdot \nabla T)$ vanishes as $(\nabla\Psi \times \hat{e}_\phi) \cdot (\nabla\Psi) = 0$
- reduces heat flux to $\vec{q} = \vec{q}_\perp = -n\chi_\perp \nabla T$
- heat diffusion equation $\nabla \cdot \vec{q} = -P \Rightarrow P = n\chi_\perp \nabla \cdot \nabla\Psi$ (to get the analytic solution)
- code runs with $\Psi_1/\Psi_0 = 10^{-2}$, $m = 3$, $n = 2$, $r_0/a = 1.2$
- resolution: $160 \times 160$ grid points
- error: $Err_{0,0} = \left[\frac{T_{num} - T_{analyt}}{T_{analyt}}\right]^{0,0}_{r=0.5a}$

# Scale island width $w_c$

- competition between parallel and perp. transport
- for $w \approx w_c$, parallel $\approx$ perpendicular transport
- scale island width $w_c = r_{res} \left(\frac{\chi_{||}}{\chi_\perp}\right)^{-1/4} \sqrt{\frac{8}{\epsilon_s s_s n}}$
  - $n$: toroidal mode number of perturbation
  - $s_s$: local magnetic shear
  - $r_{res}$: minor radius of resonant surface
  - $\epsilon_s = r_{res}/R$: local inverse aspect ratio (R: major radius)

# Finite difference scheme

- Temperature grid (blue) and heat flux grid (green).




- differential operators discretized with symm. $2^{nd}$ order form
- e.g., the radial gradient of the temperature:

$$\left[\frac{\partial T}{\partial r}\right]_{i+\frac{1}{2},j+\frac{1}{2}} = \frac{T_{i+1,j+1} + T_{i+1,j} - T_{i,j+1} - T_{i,j}}{2\Delta r} + \mathcal{O}(\Delta r^2), \quad (6)$$