Heat Diffusion Induced by Magnetic Field Perturbations

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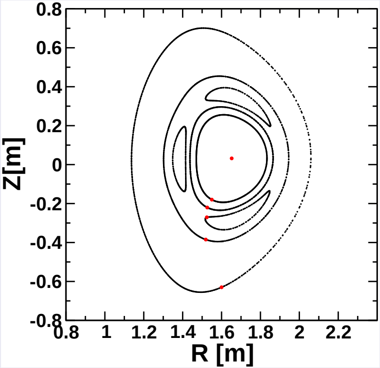
Introduction

Background

- **Helical magnetic field lines.**
- Safety factor $q = \frac{\text{toroidal turns}}{\text{poloidal turns}}$.
- Rotational transform $\iota = q^{-1}$.
- **Resonant magnetic perturbation $\Rightarrow$ magnetic islands.**

Poincaré plot



q=3/2 island

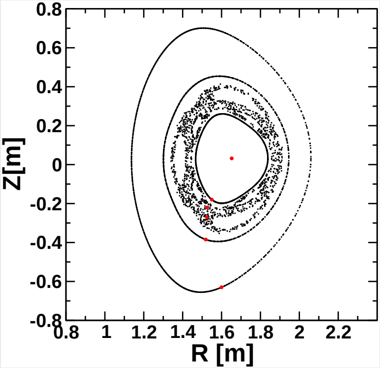
Introduction

Background

- **Helical magnetic field lines.**
- Safety factor $q = \frac{\text{toroidal turns}}{\text{poloidal turns}}$.
- Rotational transform $\iota = q^{-1}$.
- **Resonant magnetic perturbation $\Rightarrow$ magnetic islands.**
- Island overlap: **Ergodic layer.**

- **Heat diffusion across islands and ergodic layers?**

Poincaré plot



q=3/2 island + q=4/3 island

Introduction

Heat Diffusion Equation

- $$\boxed{\frac{3}{2}n\frac{\partial T}{\partial t} + \nabla \cdot \vec{q} = P}$$

- n density
- T temperature
- P power source
- $\vec{q}$ heat flux

Introduction

Heat Diffusion Equation

- $$\frac{3}{2}n\frac{\partial T}{\partial t} + \nabla\cdot\vec{q} = P$$
- $$\vec{q} = -n\chi_{||}\nabla_{||}T - n\chi_{\perp}\nabla_{\perp}T$$
- **Large anisotropy:** $\chi_{||}/\chi_{\perp} \approx 10^6 \dots 10^{10}$.
- Usually produces large **artificial diffusion** in numerical computations.
- Possibility: Exact alignment of coordinates to the magnetic field.

- n density
- T temperature
- P power source
- $\vec{q}$ heat flux
- $\nabla_{||}T = \hat{b}\left(\hat{b}\nabla T\right)$, where $\hat{b} = \vec{B}/B$
- $\nabla_{\perp}T = \nabla T - \nabla_{||}T$
- $\chi_{||}$ heat diffusivity $||$ to $\vec{B}$
- $\chi_{\perp}$ heat diffusivity $\perp$ to $\vec{B}$

Introduction

Heat Diffusion Equation

- $$\frac{3}{2} n \frac{\partial T}{\partial t} + \nabla \cdot \vec{q} = P$$
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- **Large anisotropy:** $\chi_{||}/\chi_{\perp} \approx 10^6 \ldots 10^{10}$.
- Usually produces large **artificial diffusion** in numerical computations.
- Possibility: Exact alignment of coordinates to the magnetic field.
- **Scheme found by Günter et al. (2005): only rough alignment.**

- n density
- T temperature
- P power source
- $\vec{q}$ heat flux
- $\nabla_{||} T = \hat{b}\left(\hat{b} \nabla T\right)$, where $\hat{b} = \vec{B}/B$
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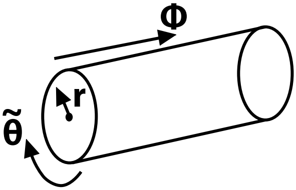
Simplified Geometry

"Periodic cylinder"

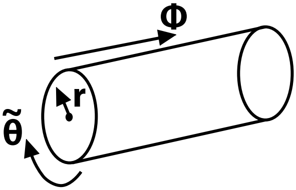
- Symmetry reduces computational effort
- Comparison to analytical theories



Simplified Geometry

"Periodic cylinder"

- Symmetry reduces computational effort
- Comparison to analytical theories
- Transformation $\theta = \tilde{\theta} - \iota_c \cdot \phi$
- **Unsheared helical coordinates** ($\iota_c = \text{const}$)
- Aligned to field lines at one surface only

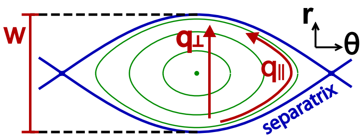


Island Heat Transport

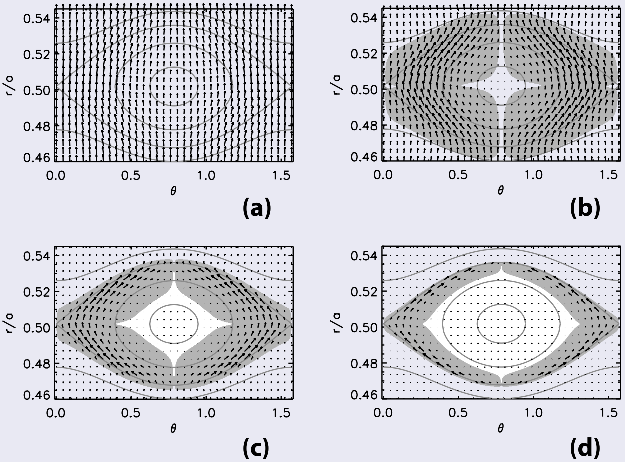
Scale island width

- **Competition** between parallel and perpendicular heat fluxes.
- If $w \gtrsim 2w_c$, parallel transport wins.
- **Scale island width** $w_c \propto \left(\frac{\chi_{\|}}{\chi_{\perp}}\right)^{-1/4}$

(Fitzpatrick, 1995)



Island Heat Transport



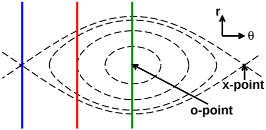
- $4/3$ island, $w = 0.068a$
- **Heat conduction layer**

	$\chi_{\|\|}/\chi_{\perp}$	w/w_c	ν
(a)	10^6	**1.1**	1
(b)	10^7	**1.9**	0.24
(c)	10^8	**3.4**	0.074
(d)	10^9	**6.0**	0.023

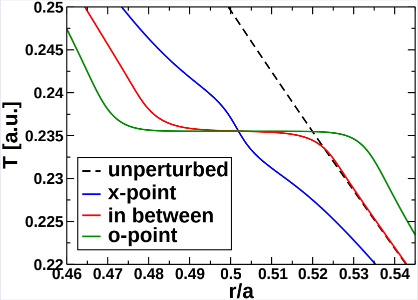
ν: vector normalization factor

(Hölzl et al., 2007)

Island Heat Transport



radial Temperature cuts



(c) $\frac{w}{w_c} = 3.4$

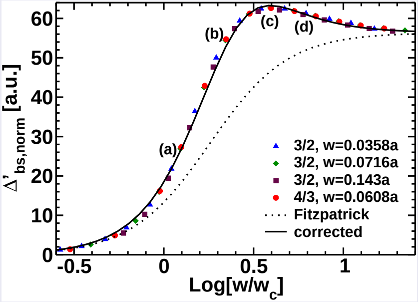
- **Temperature flattens around o-point.**

  $\Rightarrow$ Reduced bootstrap current.
- Consequence?

Neoclassical Tearing Modes

- Temperature flattening causes **effective “lack current”.**
- It **“drives” a Neoclassical Tearing Mode.**

- $\frac{\partial w}{\partial t} \propto \Delta'_{classical} + \Delta'_{bs}$
- Limits agree with analytical predictions by Fitzpatrick (1995).
- Observed **corrections up to 55%** for realistic ratios of w/w_c.



(Hölzl et al., 2007)

Realistic Geometry

Straight Field Line Coordinates ρ, $\tilde{\theta}$, ϕ

$$R = \sum_m \left[R_m(\rho) \sin(m\tilde{\theta}) + \tilde{R}_m(\rho) \cos(m\tilde{\theta}) \right]$$

$$\Phi = \phi$$

$$Z = \sum_m \left[Z_m(\rho) \sin(m\tilde{\theta}) + \tilde{Z}_m(\rho) \cos(m\tilde{\theta}) \right]$$

axisymmetric coordinates, $\Phi = \phi$

(generalisation relatively easy)

Field Aligned Coordinates ρ, θ, ϕ

$$\boxed{\theta = \tilde{\theta} - \iota_c(\rho) \cdot \phi}, \iota_c = \iota.$$

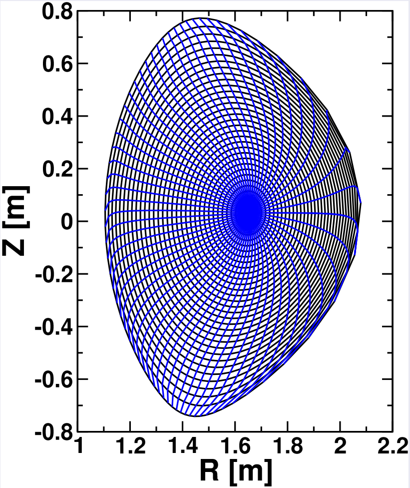
$\Rightarrow \vec{e}_\phi$ **aligned to** $\vec{B}_0$

Coordinate System

Field aligned coordinates

- Few toroidal grid points required

Grid at $\phi = 0$

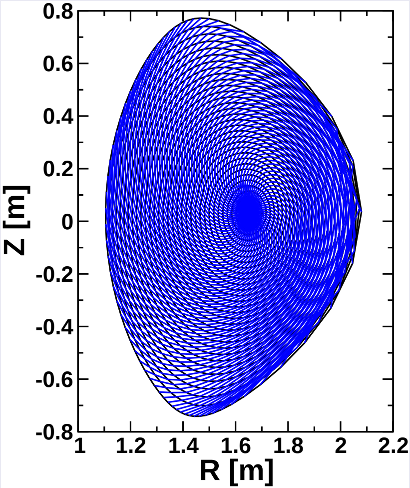


Coordinate System

Field aligned coordinates

- Few toroidal grid points required
- **Coordinates are sheared!**

Grid at $\phi = 2\pi$

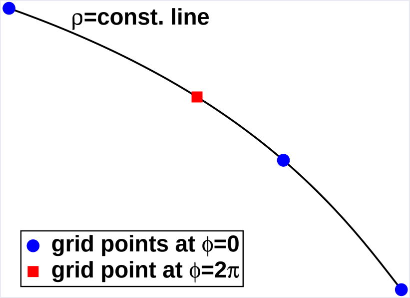


Coordinate System

Field aligned coordinates

- Few toroidal grid points required
- **Coordinates are sheared!**
- Coordinate lines do not close after one toroidal turn
- **Interpolation** for toroidal periodicity condition causes some numerical diffusion

Grid at

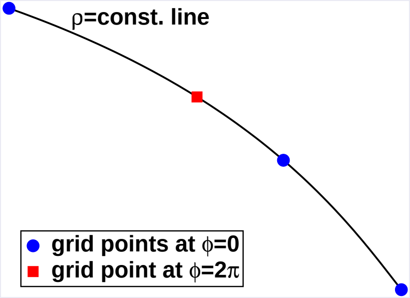


Coordinate System

Field aligned coordinates

- Few toroidal grid points required
- **Coordinates are sheared!**
- Coordinate lines do not close after one toroidal turn
- **Interpolation** for toroidal periodicity condition causes some numerical diffusion
- Alignment transformation in sections $\theta = \tilde{\theta} - \iota_c \cdot (\phi - \phi_p)$ for $\phi_p \leq \phi < \phi_{p+1}$
  $\Rightarrow$ Requires more interpolation

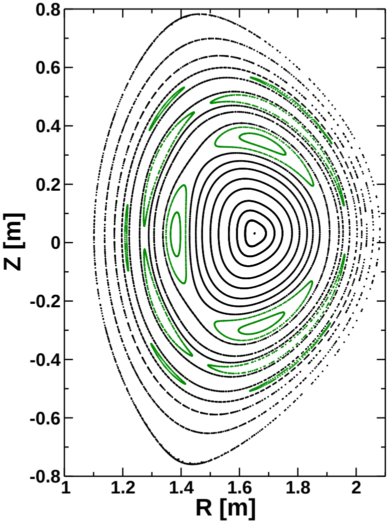
Grid at



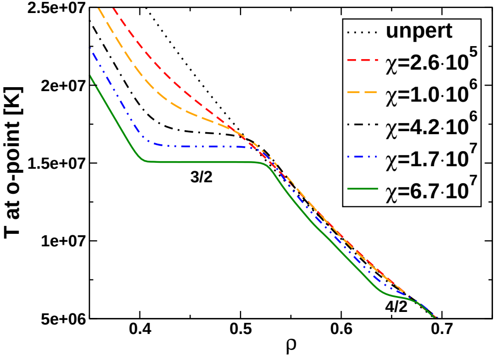
Islands in ASDEX Upgrade

- ASDEX Upgrade Equilibrium

  (close to AUG#15863@4.1s)
- External 3/2 perturbation produces **large** 3/2 **island.**
- **Higher harmonics** due to torus effects and plasma shaping.



Islands in ASDEX Upgrade



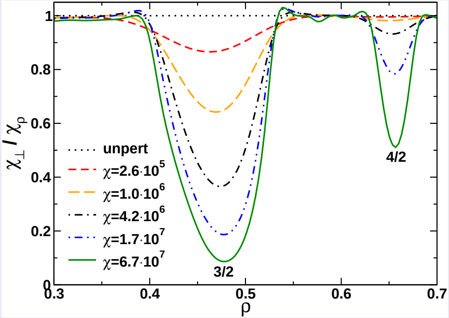
Temperature cuts through o-point of 3/2 island

- $\chi = \chi_{||}/\chi_{\perp}$
- Temperature flattening at o-point depends on χ.
- 4/2 island also flattens.

Islands in ASDEX Upgrade

Effective radial heat diffusivity χ_ρ

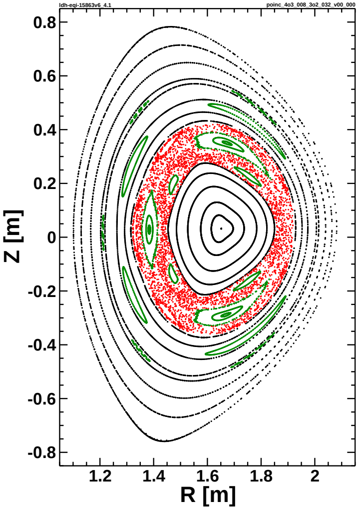
- **Inverse of radial heat diffusivity χ_ρ.**
- Radial heat diffusion is strongly increased in the island regions.



Same cases as previous slide.

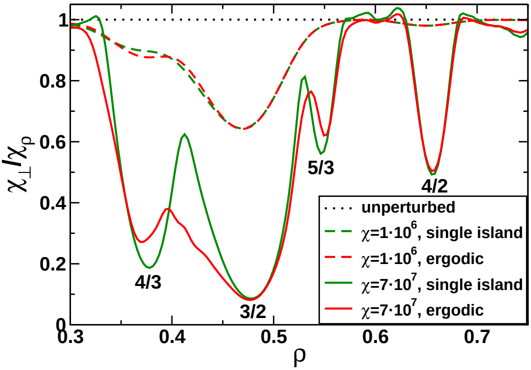
Ergodic Layer in ASDEX Upgrade

- 4/3 and 3/2 perturbations
- **Ergodic layer produced by island "overlap"**: Field line with chaotic trajectory.
- **Island remnants**



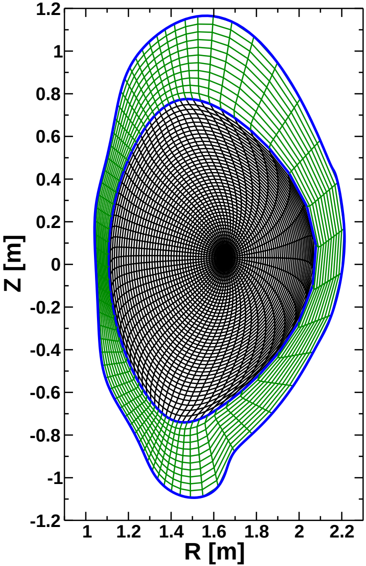
Ergodic Layer in ASDEX Upgrade

- **Inverse of the radial heat diffusion coefficient χ_ρ plotted.**
- $\chi_{||}/\chi_\perp = 1 \cdot 10^6$: Ergodic heat transport $\sim$ single island effects **(2%)**
- $\chi_{||}/\chi_\perp = 7 \cdot 10^7$: Ergodic heat transport $\neq$ single island effects **(105%)**
- **Ergodic heat transport is different from island heat transport for large $\chi_{||}/\chi_\perp$.**



Coordinate Extension

- Linear extension to first wall with continuous transition
- (Tried also quadratic, cubic, ...)
- + One coordinate system
- − Alignment error $\sim 5^\circ$; more toroidal grid points
- − $g = \det[g_{ij}]\ldots$
- Increased number of toroidal grid points required



Summary

- Heat transport in a periodic cylinder
  - Unsheared helical coordinates
  - Island temperature flattening
  - NTM stability
- Realistic geometry
  - Coordinate system aligned to $\vec{B}_0$ in plasma region
    $\Rightarrow$ Few toroidal grid points
  - Sheared helical coordinates
  - Coordinate lines do not close $\Rightarrow$ Interpolation required
  - Transformation in sections $\Rightarrow$ More interpolation
  - Island and ergodic heat transport
- Coordinate extension
  - one coordinate system
  - $\sim 5^\circ$ misalignment
  - large variation in $g = \det(g_{ij})$ values

References

Acknowledgements

- Prof. Dr. Sibylle Günter
- Dr. Qingquan Yu
- Prof. Dr. Karl Lackner
- Dr. Erika Strumberger

Codes used

- VMEC (Hirshman and Lee, 1986; Hirshman et al., 1986)
- VACFIELD (Strumberger and Schwarz, 2005)
- MFBE (Strumberger et al., 2001)
- GOURDON (Gourdon, 1970)

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Any ideas or suggestions regarding the coordinate system? Coordinate extension? Avoiding/Improving the interpolation?

Heat Diffusion Equation

- $$\boxed{\frac{3}{2}n\frac{\partial T}{\partial t} + \nabla \cdot \vec{q} = P}$$
- $$\boxed{\vec{q} = -n\chi_\perp \nabla_\perp T - n\chi_{||}\nabla_{||} T}, \chi_{||} \gg \chi_\perp$$

Tensor notation

- $\frac{3}{2}n\frac{\partial T}{\partial t} - \frac{1}{\sqrt{g}}\frac{\partial}{\partial u^\alpha}\left[C^{\alpha\beta}\frac{\partial T}{\partial u^\beta}\right] = P$,
  where $C^{\alpha\beta} = \sqrt{g} n \chi_\perp \left(\chi b^\alpha b^\beta + g^{\alpha\beta}\right)$

- n density
- T temperature
- P power source
- $\vec{q}$ heat flux
- $\vec{B}$ magnetic field
- $\hat{b} = \vec{B}/B$
- $\nabla_{||} T = \hat{b}\left(\hat{b}\nabla T\right)$
- $\nabla_\perp T = \nabla T - \nabla_{||} T$
- $\chi_{||}$ heat diffusivity $||$ to $\vec{B}$
- $\chi_\perp$ heat diffusivity $\perp$ to $\vec{B}$
- $\chi = (\chi_{||} - \chi_\perp)/\chi_\perp$
- u^α curvl. coords ρ, θ, ϕ

Straight Field Line Coordinates ρ, $\tilde{\theta}$, ϕ

$$R = \sum_{m,n} \left[R_{m,n}(\rho) \sin(m\tilde{\theta} + n\phi) + \tilde{R}_{m,n}(\rho) \cos(m\tilde{\theta} + n\phi) \right]$$

$$\Phi = \phi + \sum_{m,n} \left[V_{m,n}(\rho) \sin(m\tilde{\theta} + n\phi) + \tilde{V}_{m,n}(\rho) \cos(m\tilde{\theta} + n\phi) \right]$$

$$Z = \sum_{m,n} \left[Z_{m,n}(\rho) \sin(m\tilde{\theta} + n\phi) + \tilde{Z}_{m,n}(\rho) \cos(m\tilde{\theta} + n\phi) \right]$$

Field Aligned Coordinates ρ, θ, ϕ

$$\boxed{\theta = \tilde{\theta} - \iota_c(\rho) \cdot (\phi - \phi_p)}$$

where $\iota_c = 0$, $\iota_c = \text{const}$, or $\iota_c = \iota$. **In sections $\phi_p \leq \phi < \phi_{p+1}$.**