The non-linear MHD Code JOREK

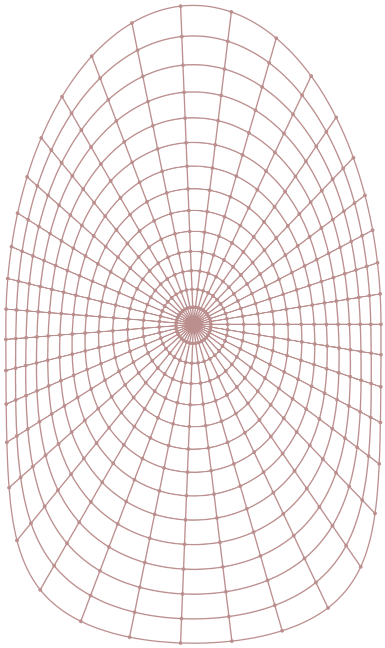
IPP Ringberg Theory Meeting 2010

Matthias Hölzl

JOREK

- 3D non-linear MHD code (divertor tokamaks)
- Developed by Guido Huysmans at CEA Cadarache
- Main target: ELM simulations
- 2D Bezier finite elements
- Toroidal Fourier-decomposition
- Fully implicit time-stepping
- Currently reduced MHD
- MPI + OpenMP (ELM simulation: $\geqslant$ 256 CPUs)

[G Huysmans et al, Plasma Phys Control Fusion 51, 124012 (2009)]

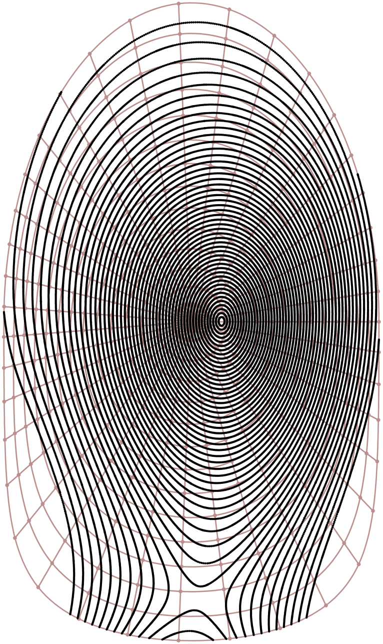


Typical code run

- Initial grid
- Grad-Shafranov equation

$$\Delta^*\Psi = -\mu_0 R^2 p'(\Psi) - \mu_0^2 f(\Psi) f'(\Psi)$$

  - toroidal field
  - pressure profile
  - plasma current profile
  - poloidal flux at boundary
- Flux-surface grid
- Grad-Shafranov equation
- Equilibrium refinement
- Time-integration



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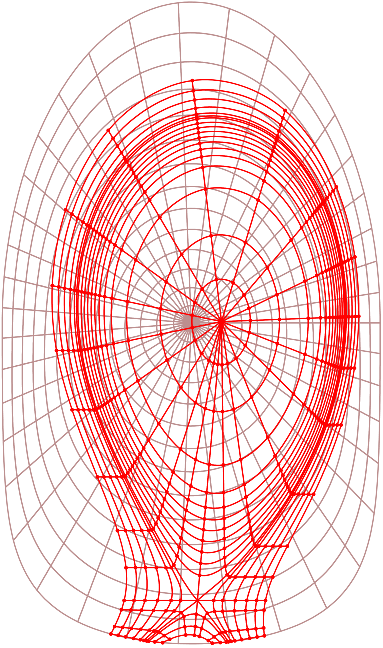

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Reduced MHD Equations

$$\mathbf{B} = \frac{F_0}{R}\mathbf{e}_\phi + \frac{1}{R}\nabla\Psi \times \mathbf{e}_\phi \qquad j = \Delta^*\Psi$$

$$\mathbf{v} = -R\nabla u \times \mathbf{e}_\phi + v_{||}\mathbf{B} \qquad \omega = \nabla^2_{pol}\, u \qquad p = \rho T$$

$$\frac{1}{R^2}\frac{\partial\Psi}{\partial t} = \eta(T)\ \nabla\cdot\left(\frac{1}{R^2}\nabla_\perp\Psi\right) - \frac{1}{R}[u,\Psi] - \frac{F_0}{R^2}\frac{\partial u}{\partial\phi}$$

$$\mathbf{e}_\phi\cdot\nabla\times\left[\rho\frac{\partial\mathbf{v}}{\partial t}\right] = \mathbf{e}_\phi\cdot\nabla\times[-\rho(\mathbf{v}\cdot\nabla)\mathbf{v} - \nabla(\rho T) + \mathbf{J}\times\mathbf{B} + \mu\Delta\mathbf{v}]$$

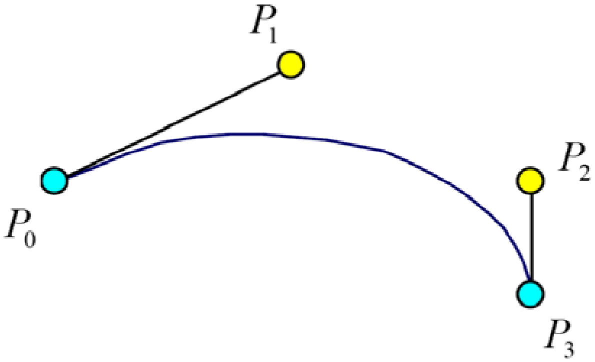
$$\frac{\partial\rho}{\partial t} = -\nabla\cdot(\rho\mathbf{v}) + \nabla\cdot(D_\perp\nabla_\perp\rho) + S_\rho$$

$$\rho\frac{\partial T}{\partial t} = -\rho\mathbf{v}\cdot\nabla T - (\gamma-1)\rho T\nabla\cdot\mathbf{v} + \nabla\cdot(K_\perp\nabla_\perp T + K_{||}\nabla_{||}T) + S_T$$

$$\mathbf{B}\cdot\left[\rho\frac{\partial\mathbf{v}}{\partial t}\right] = \mathbf{B}\cdot[-\rho(\mathbf{v}\cdot\nabla)\mathbf{v} - \nabla(\rho T) + \mathbf{J}\times\mathbf{B} + \mu(T)\Delta\mathbf{v}]$$

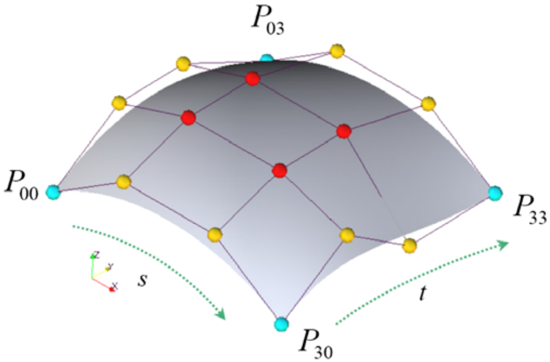
- $\mathcal{O}(\epsilon^2)$ equations, where $\epsilon = a/R_0$; different models
- The variables Ψ, u, ρ, T, and $v_{||}$ are time-integrated (plus j- and ω-equations)

Bezier Elements



[O Czarny and G Huysmans, J Comput Phys 227, 7423 (2008)]

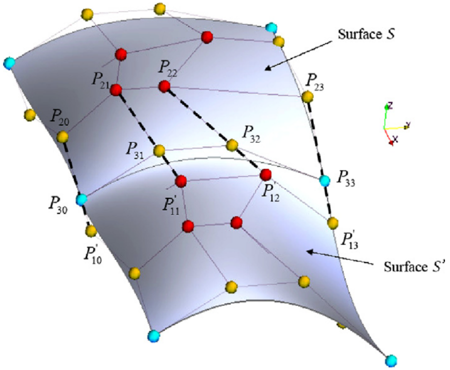
Bezier Elements



- 2D Bezier finite elements
- Isoparametric

[O Czarny and G Huysmans, J Comput Phys 227, 7423 (2008)]

Bezier Elements



- 2D Bezier finite elements
- Isoparametric
- C^1 continuity $\Rightarrow$ Four degrees of freedom per node and variable

[O Czarny and G Huysmans, J Comput Phys 227, 7423 (2008)]

Time Stepping

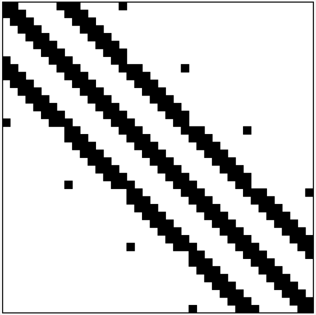
- Linearized Crank-Nicholson

[R M Beam and R F Warming, SIAM Journal on Scientific and Statistical Computing 1, 131 (1980)] [C Hirsch (1991), ISBN 9780471923855]

$$\frac{\partial \mathbf{A}(\mathbf{u})}{\partial t} = \mathbf{B}(\mathbf{u}, t) \quad \rightarrow \quad \left[\left(\frac{\partial \mathbf{A}}{\partial \mathbf{u}} \right)^n - \frac{\Delta t}{2} \left(\frac{\partial \mathbf{B}}{\partial \mathbf{u}} \right)^n \right] \delta \mathbf{u}^n = \Delta t \; \mathbf{B}^n$$

$$\delta \mathbf{u}^n = \mathbf{u}^{n+1} - \mathbf{u}^n$$

- Fully implicitly (all equations solved simultaneously)



- Sparse matrix system
- Dense blocks
- Block size: $n_{\text{tor}} \times n_{\text{var}} \times n_{\text{dof}}$

(Example: low resolution, no X-point)

Time Stepping

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- PASTIX (Parallel Sparse Matrix Package; direct)

  [Hénon et al, Parallel Comput, 28, 301 (2002), http://dx.doi.org/10.1016/S0167-8191(01)00141-7]

- HIPS (Hierarchical Iterative Parallel Solver; direct+ILU)

  [J Gaidamour and P Henon, Proceedings PMAA 2008, http://www.cerfacs.fr/algor/PastWorkshops/SparseDays2008/Slides/henon.pdf]

- MURGE (common interface to PASTIX and HIPS)

  [https://gforge.inria.fr/projects/murge]

- GMRES (Generalized Minimal Residual Method; iterative)
  with PASTIX preconditioner (independent solution for toroidal harmonic)

  [V. Frayss et al, Technical Report TR/PA/06/09, CERFACS, Toulouse, France, http://www.cerfacs.fr/algor/reports/2006/TR_PA_06_09.pdf]

Discretization of physical equations

- Time integration (implemented more generally)

$$\frac{\partial \mathbf{A}(\mathbf{u})}{\partial t} = \mathbf{B}(\mathbf{u}, t) \quad \rightarrow \quad \left[\left(\frac{\partial \mathbf{A}}{\partial \mathbf{u}} \right)^n - \frac{\Delta t}{2} \left(\frac{\partial \mathbf{B}}{\partial \mathbf{u}} \right)^n \right] \delta \mathbf{u}^n = \Delta t \mathbf{B}^n$$

- Weak formulation, Partial integration

$$\int_V dV \, v \, (\dots)$$

- Integration by Gauss Quadrature

$$\int_0^1 ds \, A(s) = \sum_{\mu=1}^{4} w_\mu \, A(s_\mu) \quad (w_\mu \text{ and } s_\mu\text{: weights and positions})$$

- Discretization of operators, e.g.,

$$[a, b] = a_{,R} b_{,Z} - a_{,Z} b_{,R} \quad \text{with} \quad a_{,R} = (a_{,s} Z_{,t} - a_{,t} Z_{,s}) / (R_{,s} Z_{,t} - R_{,t} Z_{,s})$$

- Expansion of variables in node degrees of freedom

Activities in TOK

- Run the code
  RZG, M. Hölzl, W.-C. Müller
- Understand the code
  M. Hölzl, W.-C. Müller
- Write a documention
  M. Hölzl
- Improve diagnostics
  M. Hölzl
- AUG equilibria
  M. Hölzl, E. Strumberger
- Sheared rotation
  M. Hölzl, W.-C. Müller
- Free boundary
  P. Merkel, G. Huysmans, M. Hölzl
- Tearing modes
  M. Hölzl, I. Krebs, W.-C. Müller
- To be done:
  ELMs, Vertical Stability, . . .

Documentation

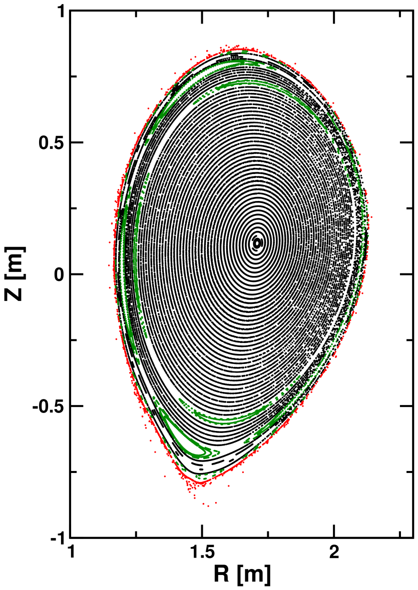
- Currently no real documentation
- Writing down important information successively, e.g.,
  - Coordinate systems and discretization
  - Compiling and running the code
  - Code structure
  - Implementing equations in JOREK
  - Physical equations
  - Time integration
- Currently 50 pages, available on the subversion server

Diagnostics

- Poincaré plots
- Connection length
- Visualization with VISIT/Paraview (.vtk files)

New

- Poincaré plots vs. Ψ_n-θ
- Determination of θ_{mag} (to be improved)
- 2D Fourier analysis using FFTW
- Parallel batch conversions
- Structured HTML job report



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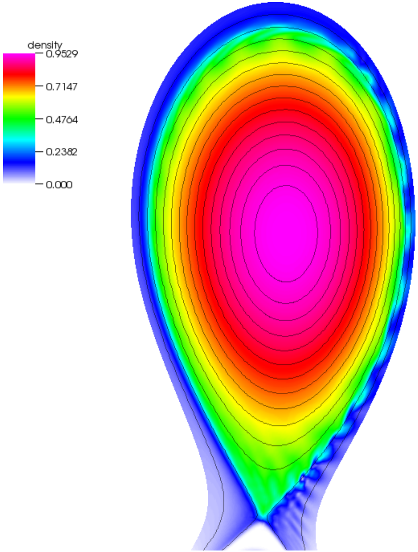

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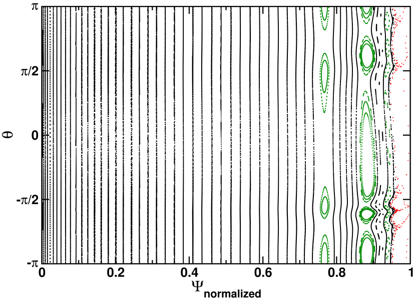

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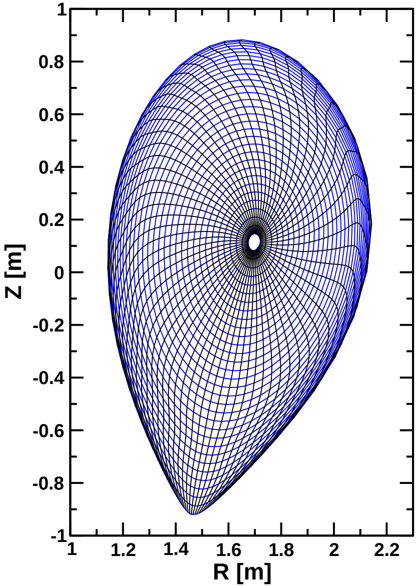

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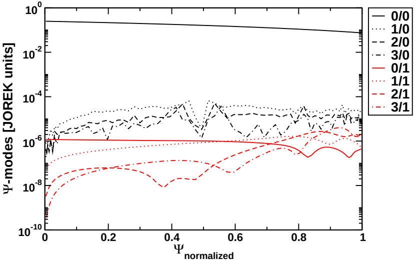

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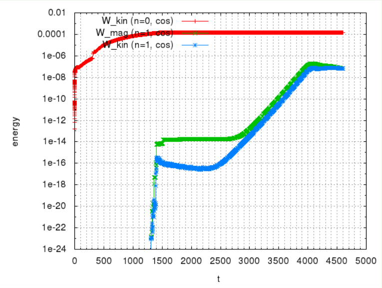
[+] **ERRORS AND WARNINGS**

[+] **README FILES**

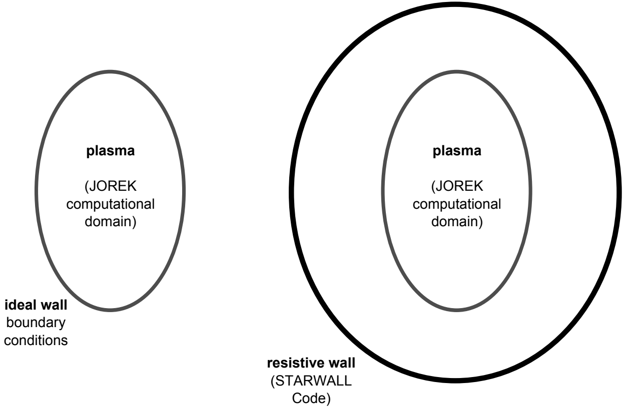
[+] **readme**

```
restart of run1b_noinit_source at step 390 with u-source increased by factor 5
```

[+] **ENERGY DIAGNOSTIC**



Resistive Walls: Status and Plans



Physics aims

- Resistive Wall Modes
- Vertical Instabilities
- Disruptions

Resistive Walls: Coupling JOREK and STARWALL

Changes in JOREK

- Equilibrium determination
- Time-evolution of wall current potentials, Y:

$$\dot{Y} = -\frac{1}{\sigma d} \underbrace{D^{-1}} Y - \underbrace{D^{-1}\ S^T\ \tilde{M}_{we}} \dot{\Psi}$$

- Boundary integral $\int dA\ \nu\ (\mathbf{B} \times \mathbf{n}) \cdot \mathbf{e}_\phi$ in current equation $j = \Delta^* \Psi$ (from partial integration)

$$(\mathbf{B} \times \mathbf{n}) \cdot \mathbf{e}_\phi = \underbrace{\tilde{M}_{||,w}\ S} Y + \underbrace{\tilde{M}_{||,e}} \Psi + \underbrace{C}$$

- Matrices computed by STARWALL

Involved: Guido Huysmans, Peter Merkel, Matthias Hölzl

Resistive Walls: Status and Plans

Status

- Free-boundary equilibrium implemented
  - Some numerical details to be solved
- Boundary integral in current equation
  - $n = 1$ ideal wall response successfully benchmarked with CASTOR
  - $n = 1$ resistive wall response almost finished

To be done

- Special cases for X-point grid (corners)
- Coupling of harmonics (3D wall)
- Call STARWALL directly from JOREK
- Clean up implementation

Simulations

Aim

- Realistic ASDEX Upgrade simulations (ELMs, tearing modes, . . .)
- CLISTE equilibria, but pressure profile, e.g., from AUGPED

Equilibria

- VMEC/NEMEC (semi-automatically)
- Numerical representation of $\mathtt{ff}'$-profiles
- Works essentially, still some deviations

Plasma rotation

- Weak rotation $\Rightarrow$ Strong mode coupling
- Implemented poloidal plasma rotation
  - Initial condition
  - Source term

1. The JOREK Code
2. Ongoing Activities
3. Summary

Summary

JOREK

- Physical model (reduced MHD)
- Spatial discretization (Bezier elements)
- Time integration scheme (fully implicit)
- Sparse matrix solvers (iterative with physics-based preconditioner)

Ongoing Activities

- Documentation
- Diagnostics
- Free boundary (RWMs, vertical stability, disruptions)
- AUG equilibria (tearing modes, ELMs)
- Poloidal rotation

Thanks for your attention!

Acknowledgements

Guido Huysmans

Wolf-Christian Müller

Sibylle Günter

Peter Merkel

Erika Strumberger

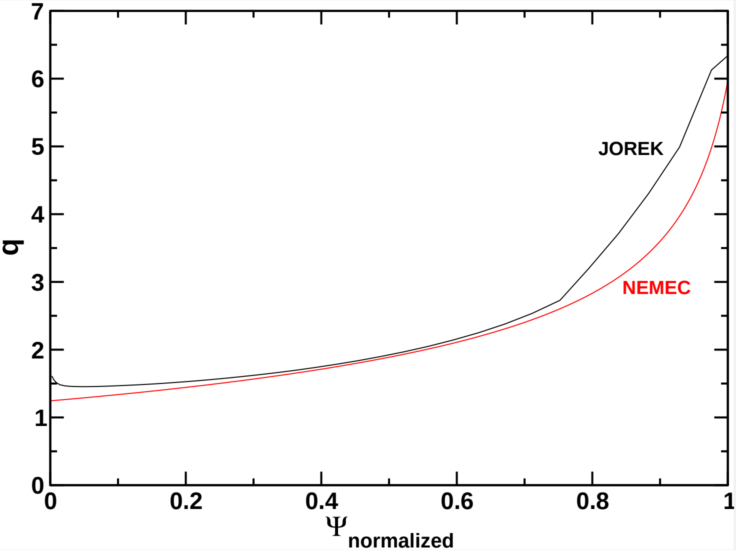
Isabel Krebs

Selected References

O Czarny and G Huysmans, J Comput Phys 227, 7423 (2008)

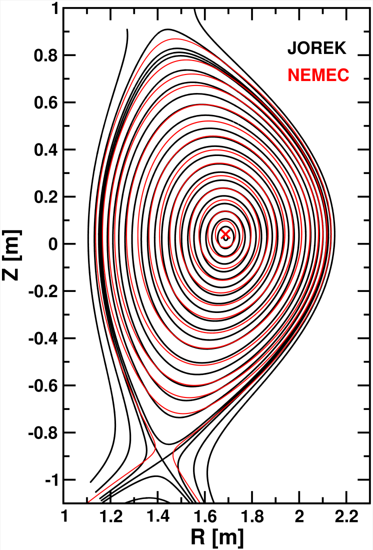
G Huysmans et al, Plasma Phys Control Fusion 51, 124012 (2009)

NEMEC interface



LDH/EQB/15863/4.1s

NEMEC interface



LDH/EQB/15863/4.1s

$$\mathbf{B} = \frac{F_0}{R}\mathbf{e}_\phi + \frac{1}{R}\nabla\Psi \times \mathbf{e}_\phi$$

- Field parallel to the interface:

$$\begin{aligned}
[\mathbf{B} \times \mathbf{n}] \cdot \mathbf{e}_\phi &= \left[\left(\frac{F_0}{R}\mathbf{e}_\phi + \frac{1}{R}\nabla\Psi \times \mathbf{e}_\phi\right) \times \mathbf{n}\right] \cdot \mathbf{e}_\phi \\
&= \left[\mathbf{e}_\phi \left(\mathbf{n} \cdot \frac{1}{R}\nabla\Psi\right) - \frac{1}{R}\nabla\Psi \left(\mathbf{n} \cdot \mathbf{e}_\phi\right)\right] \cdot \mathbf{e}_\phi \\
&= \frac{1}{R}\ \nabla\Psi \cdot \mathbf{n}
\end{aligned}$$

- Field perpendicular to the interface:

$$\begin{aligned}
\mathbf{B} \cdot \mathbf{n} &= \left[\frac{F_0}{R}\mathbf{e}_\phi + \frac{1}{R}\nabla\Psi \times \mathbf{e}_\phi\right] \cdot \mathbf{n} \\
&= -\frac{1}{R}\left(\nabla\Psi \times \mathbf{n}\right) \cdot \mathbf{e}_\phi
\end{aligned}$$

Who's working on/with JOREK currently?

Stanislas Pamela	`Stanislas.Pamela@jet.uk`	JET, ELMs
Pierre Ramet	`ramet@labri.fr`	Pastix
Xavier Lacoste	`lacoste@labri.fr`	PastiX, Murge, Makefile
Florent Sourbier	`florent.sourbier@cea.fr`	HPCFF support team
Marina Becoulet	`Marina.becoulet@cea.fr`	neoclassical flow, RMP
Virginie Grandgirard	`Virginie.GRANDGIRARD@cea.fr`	numerics
Guillaume Latu	`guillaume.latu@cea.fr`	parallelisation
Boniface Nkonga	`Boniface.Nkonga@unice.fr`	full MHD, code platform
Herve Guillard	`Herve.Guillard@sophia.inria.fr`	turbulence
Matthias Hoelzl	`mhoelzl@ipp.mpg.de`	vacuum, ELMs, tearing modes
Egbert Westerhof	`E.Westerhof@rijnhuizen.nl`	tearing modes, ECRH, FP
Ian Chapman	`ian.chapman@ccfe.ac.uk`	resistive wall modes
Guido Huysmans	`guido.huysmans@cea.fr`	pellets, numerics, vacuum

Guido Huysmans, 09/2010