Coupling JOREK and STARWALL Codes for Nonlinear Resistive Wall Simulations

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Introduction

Plasma instabilities like vertical displacement events, disruptions, external kink modes, or edge localized modes induce mirror currents in conducting structures which act back onto the instabilities and may significantly influence their linear and non-linear dynamics. We describe the present status of a resistive wall extension for the non-linear MHD code JOREK [1, 4].

STARWALL

STARWALL solves the vacuum magnetic field equation outside the JOREK computational domain in presence of a three-dimensional conducting wall with holes as a Neumann-like problem. Currents are assumed constant within each wall triangle such that they can be expressed by potentials Y_k at the triangle corners [6]. The magnetic field tangential to the inteface is given by

$$B_{\tt tan} = \sum_i b_i \left(\sum_j \hat{M}^{ee}_{i,j} \cdot \Psi_j + \sum_k \hat{M}^{ey}_{i,k} \cdot Y_k \right) \quad (1)$$

and wall currents evolve in time according to

$$\dot{Y}_k = -\frac{\eta_{\rm w}}{d_{\rm w}} \hat{M}^{yy}_{k,k}\, Y_k - \sum_j \hat{M}^{ye}_{k,j}\, \dot{\Psi}_j. \quad (2)$$

Some information about STARWALL can be found in Refs. [6, 7]. An article describing more details is in preparation by code author Peter Merkel.

Time-Discretization

Equation (1) is evaluated at timestep $n+1$ and discretizations $\Psi^{n+1} = \Psi^n + \delta\Psi^n$ and $Y^{n+1} = Y^n + \delta Y^n$ are used. Thus, the tangential field is given by

$$B^{n+1}_{\tt tan} = \sum_i b_i \left[\sum_j \hat{M}^{ee}_{i,j} \cdot (\Psi^n_j + \delta\Psi^n_j) + \sum_k \hat{M}^{ey}_{i,k} \cdot (Y^n_k + \delta Y^n_k) \right]. \quad (3)$$

The wall-current evolution (Eq. (2)) is discretized in time according to the time-evolution scheme described in Ref. [3] which is also used for the other JOREK equations. After solving for δY^n_k, one gets

$$\delta Y^n_k = \sum_j \hat{A}_{k,j}\, \delta\Psi^n_j + \hat{B}_{k,k}\, Y^n_k + \hat{C}_{k,k}\, \delta Y^{n-1}_k + \sum_j \hat{D}_{k,j}\, \delta\Psi^{n-1}_j, \quad (4)$$

where some of the "derived response matrices"

$$\begin{array}{lll} \hat{S}_{k,k} = 1 + \xi + \Delta t\theta \frac{\eta_{\rm w}}{d_{\rm w}} \hat{M}^{yy}_{k,k} & \hat{D}_{k,j} = \xi \hat{M}^{ye}_{k,j}/\hat{S}_{k,k} & \hat{H}_{i,j} = \hat{M}^{ee}_{i,j} \\ \hat{A}_{k,j} = -(1+\xi)\, \hat{M}^{ye}_{k,j}/\hat{S}_{k,k} & \hat{E}_{i,j} = \hat{M}^{ee}_{i,j} + \sum_k \hat{M}^{ey}_{i,k}\, \hat{A}_{k,j} & \hat{J}_{i,j} = \sum_k \hat{M}^{ey}_{i,k}\, \hat{D}_{k,j} \\ \hat{B}_{k,k} = -\Delta t \frac{\eta_{\rm w}}{d_{\rm w}} \hat{M}^{yy}_{k,k}/\hat{S}_{k,k} & \hat{F}_{i,k} = \hat{M}^{ey}_{i,k}(1 + \hat{B}_{k,k}) & \hat{K}_{k,l} = -\Delta t \hat{M}^{yy}_{k,k} \hat{S}_{l,k} \\ \hat{C}_{k,k} = \xi/\hat{S}_{k,k} & \hat{G}_{i,k} = \hat{M}^{ey}_{i,k}\, \hat{C}_{k,k} & \hat{L}_{i,l} = \sum_k \hat{M}^{ey}_{i,k}\, \hat{K}_{k,l} \end{array} \quad (5)$$

have been used. Plugging Eq. (4) into Eq. (3), we get

$$B^{n+1}_{\tt tan} = \sum_i b_i \left[\sum_j \hat{E}_{i,j}\, \delta\Psi^n_j + \sum_k \hat{F}_{i,k}\, Y^n_k + \sum_k \hat{G}_{i,k}\, \delta Y^{n-1}_k + \sum_j \hat{H}_{i,j}\, \Psi^n_j + \sum_j \hat{J}_{i,j}\, \delta\Psi^{n-1}_j \right]. \quad (6)$$

Coupling via Natural Boundary Condition

The current definition $j = \Delta^*\Psi \equiv R^2\, \nabla \cdot (R^{-2}\, \nabla\Psi)$ can be written in weak form as

$$\int dV\, \frac{j^*_l}{R^2}\, j - \int dV\, j^*_l\, \nabla \cdot \left(\frac{1}{R^2} \nabla\Psi \right) = 0, \quad (7)$$

where the second term may be integrated by parts yielding

$$\int dV\, \frac{j^*_l}{R^2}\, j + \int dV\, \frac{1}{R^2}\, \nabla j^*_l \cdot \nabla\Psi - \oint dA\, \frac{j^*_l}{R}\, \underbrace{(\nabla\Psi \cdot \hat{\mathbf{n}}/R)}_{\equiv B_{\tt tan}} = 0. \quad (8)$$

Inserting Eq. (6) into the boundary integral and separating implicit and explicit terms yields the form implemented in JOREK,

$$\begin{aligned} &\sum_{i_{\rm elem}} \int \frac{dV}{R^2} \left(j^*_l\, \delta j^n + \nabla j^*_l \cdot \nabla\delta\Psi^n \right) - \sum_{i_{\rm bnd}} \oint dA \frac{j^*_l}{R} \sum_i b_i \sum_j \hat{E}_{i,j}\, \delta\Psi^n_j \\ &= -\sum_{i_{\rm elem}} \int \frac{dV}{R^2} \left(j^*_l\, j^n + \nabla j^*_l \cdot \nabla\Psi^n \right) \\ &\quad + \sum_{i_{\rm bnd}} \oint dA \frac{j^*_l}{R} \sum_i b_i \left[\sum_k \left(\hat{F}_{i,k}\, Y^n_k + \hat{G}_{i,k}\, \delta Y^{n-1}_k \right) + \sum_j \left(\hat{H}_{i,j}\, \Psi^n_j + \hat{J}_{i,j}\, \delta\Psi^{n-1}_j \right) \right]. \end{aligned} \quad (9)$$

Wall-currents are updated after each time-step according to Eq. (4) to guarantee consistency with the implicit time-stepping of JOREK. For the $n = 0$ component, poloidal field coils need to be taken into account additionally, which leads to two additional terms in Eq. (9). A similar boundary integral occurs in the Grad-Shafranov equation for the plasma equilibrium.

First Benchmarks and Results

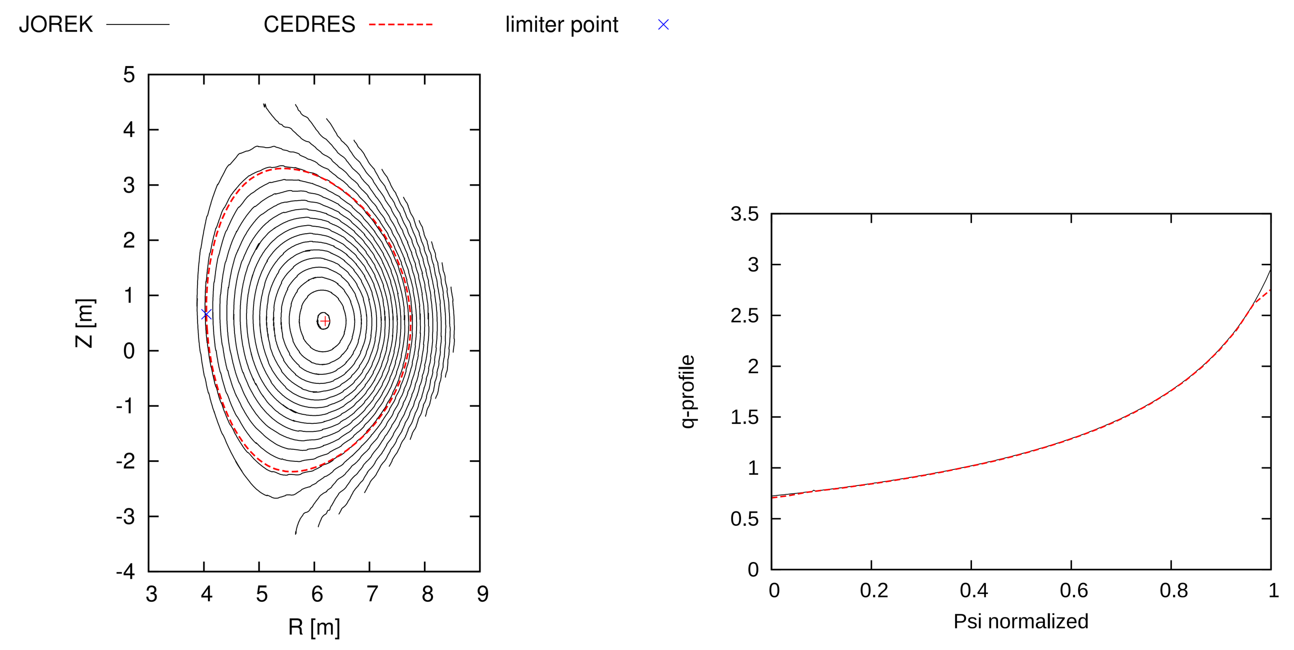

Figure : For an ITER-like limiter case, the freeboundary equilibrium determined by JOREK+STARWALL is compared to the results from the CEDRES++ code [2]. Very good agreement is observed with the small remaining differences caused by different coil discretizations.

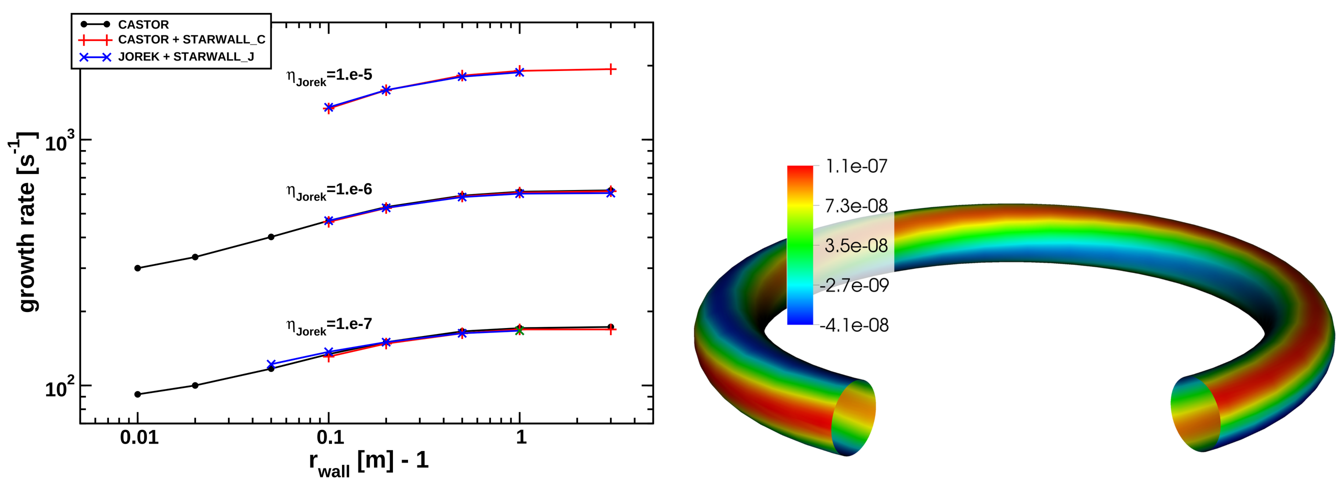

Figure : For a $2/1$ tearing mode in a circular plasma surrounded by an ideally conducting wall, the linear growth rates obtained from the JOREK+STARWALL simulations are compared to results from the linear CASTOR code [5].

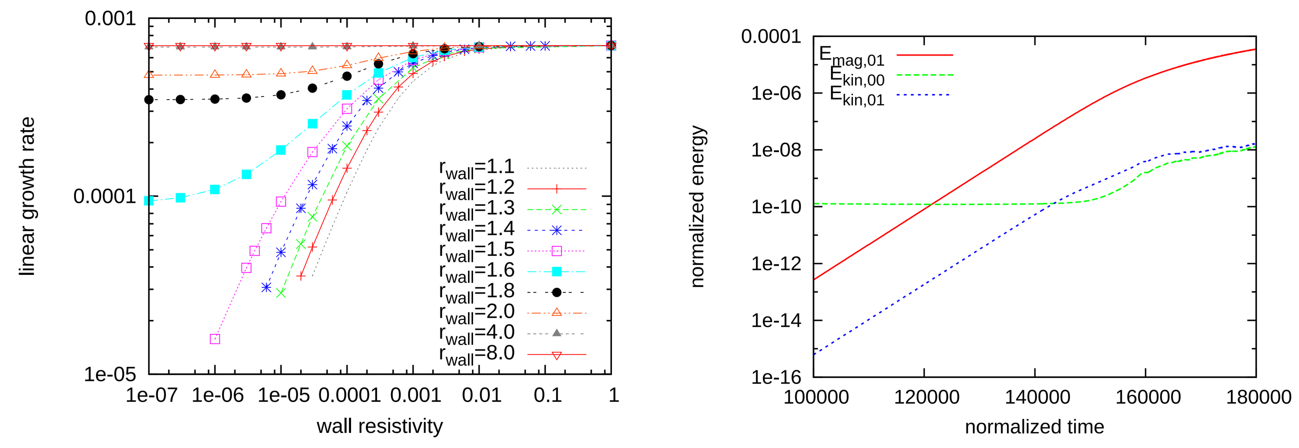

Figure : For a resistive wall mode in a circular plasma surrounded by a resistive wall, the linear growth rates are plotted as a function of the wall radius. Also, energy time-traces during non-linear saturation are shown. A comparison to analytical theory is in preparation.

Summary and Outlook

The ongoing implementation and verification of a resistive wall model in the non-linear MHD-code JOREK was summarized. Benchmarks for a freeboundary equilibrium and tearing mode cases show good agreement. Simulations of the linear and non-linear phase of RWMs were presented (comparison to analytical theory and linear codes ongoing).
Benchmarking will be continued and extended to non-linear comparisons.
Realistic X-point geometries will be considered requiring a special treatment of grid corners. After completion, the code can be applied to a variety of MHD instabilities interacting with conducting structures like resistive wall modes, edge localized modes, vertical displacement events or disruptions.

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