Nonlinear Drive of Low-n Toroidal Harmonics in Reduced MHD Simulations of ELMs

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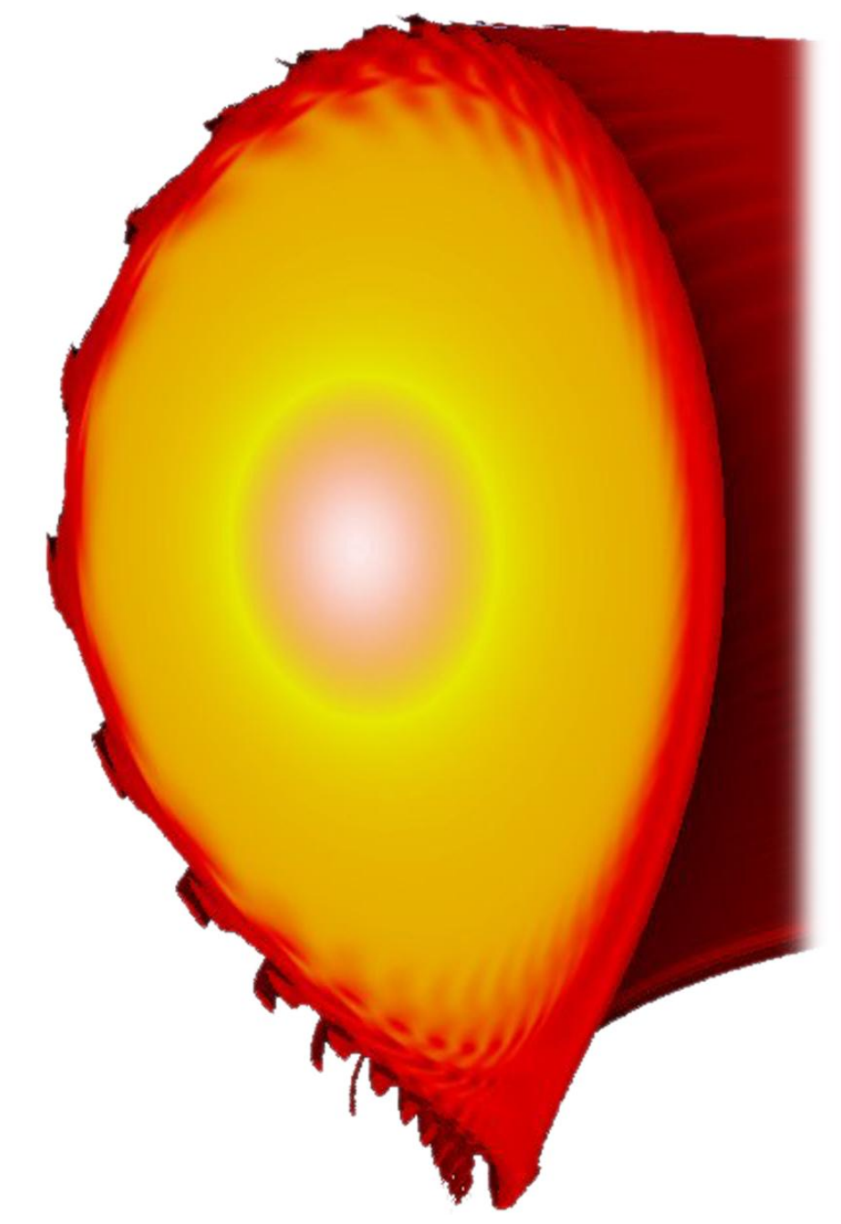
Abstract

Nonlinear simulations of the early edge-localized mode (ELM) phase based on a typical type-I ELMy ASDEX Upgrade tokamak discharge have been carried out using the reduced MHD code JOREK. A large number of toroidal Fourier harmonics has been included in the simulations and the analysis has been focused on their nonlinear interaction. During the early nonlinear ELM phase, linearly subdominant low-n harmonics, in particular the n=1, grow to energies comparable with linearly dominant harmonics. The nonlinear evolution of the toroidal Fourier spectrum in the JOREK simulations is reproduced and explained very well by a simple model which is based on the idea, that energy is transferred among the toroidal harmonics via second order nonlinear coupling. This quadratic interaction model might also explain the recent experimental observation of strong low-n components in ELMy tokamak discharges [1]. Furthermore, the spatial structure of the n=1 harmonic during its nonlinear evolution in the simulations is studied. Whereas the structure of the linearly barely unstable n=1 extends over a large part of the plasma core, the nonlinearly excited n=1 is localized at the plasma edge, where the dominant toroidal harmonics driving the n=1 are maximal and in phase. [2]



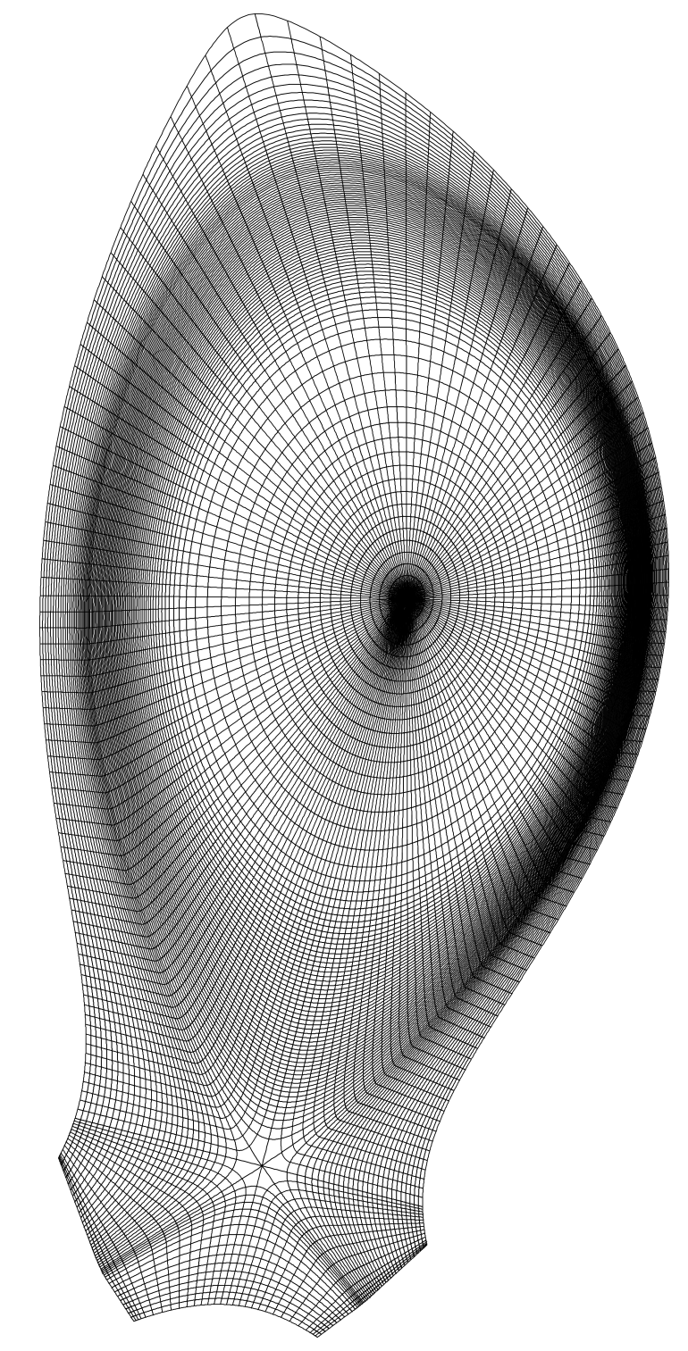
Edge-localized modes

- relaxation-oscillation instability in tokamak plasmas
- occur at the edge of plasma in high confinement mode (H-mode)
- driven by large pressure gradients & current densities
- each event ejects particles & energy from the plasma

- **advantage:** help to control particle & impurity content
- **problem:** high heat fluxes can damage plasma facing components

The JOREK code

- solves nonlinear reduced MHD equations
- full toroidal X-point geometry including separatrix & open field lines
- 2D Bezier finite elements in poloidal plane & toroidal Fourier decomposition
- flux surface aligned grid
- fully implicit time stepping
- originally developed by G.T.A. Huysmans [3,4]

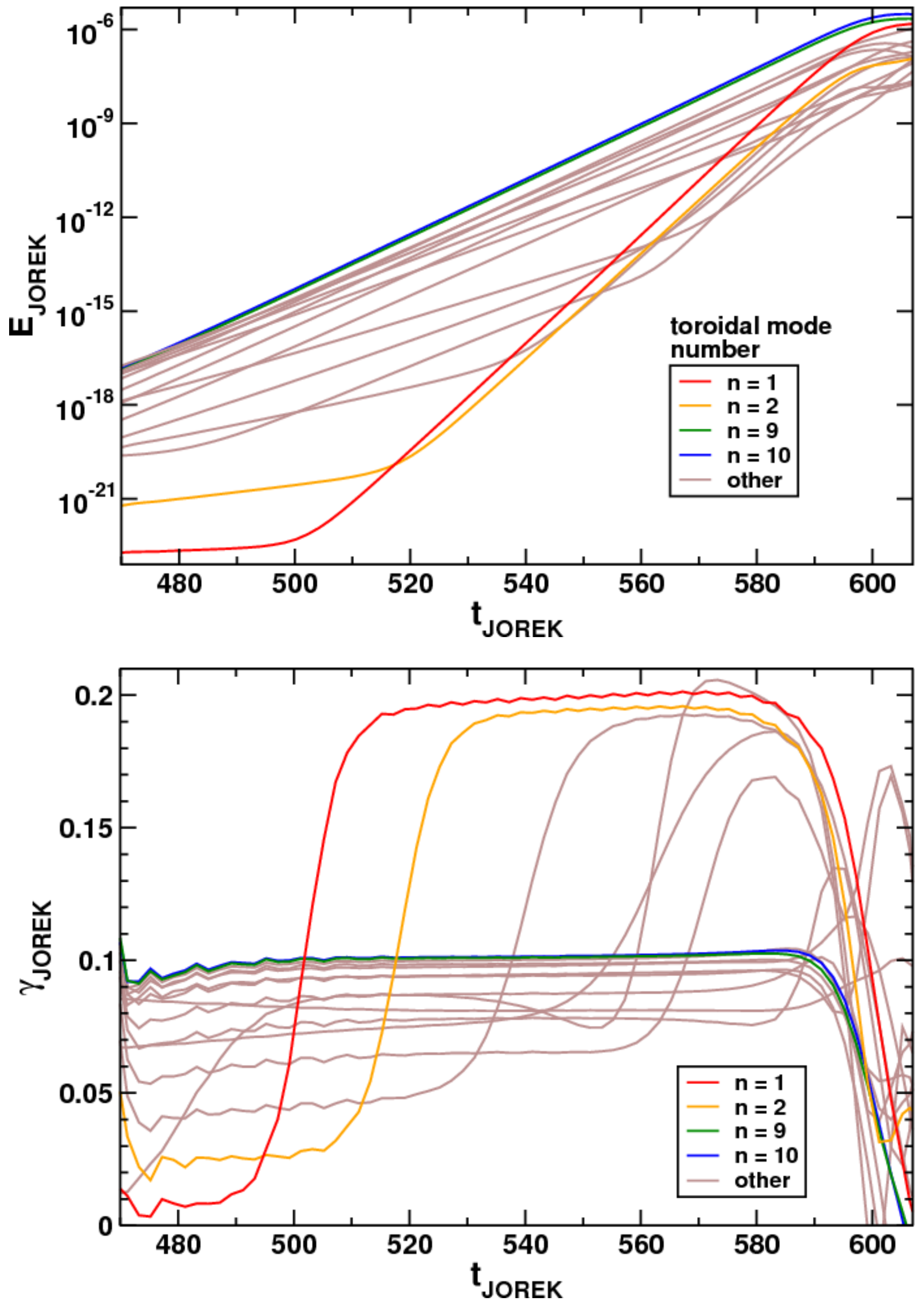


The simulations

- early ELM phase until onset of nonlinear saturation (full crash simulations also available [5])
- large number of included toroidal Fourier harmonics (n=1,2,...,16 and various subsets)
- based on typical type-I ELMy H-mode discharge of the ASDEX Upgrade tokamak
- realistic viscosity and heat diffusion anisotropy, increased resistivity for computational reasons
- ideally conducting wall boundary conditions

- **analysis:** Fourier decomposition of perturbation

$$\xi(x) = \sum_{m,n=-\infty}^{+\infty} \xi_{m,n}(r) \exp[-i(m\theta + n\phi)]$$



Energies and growth rates of the toroidal Fourier harmonics in the early ELM phase of a simulation with n=1, 2, ..., 16.

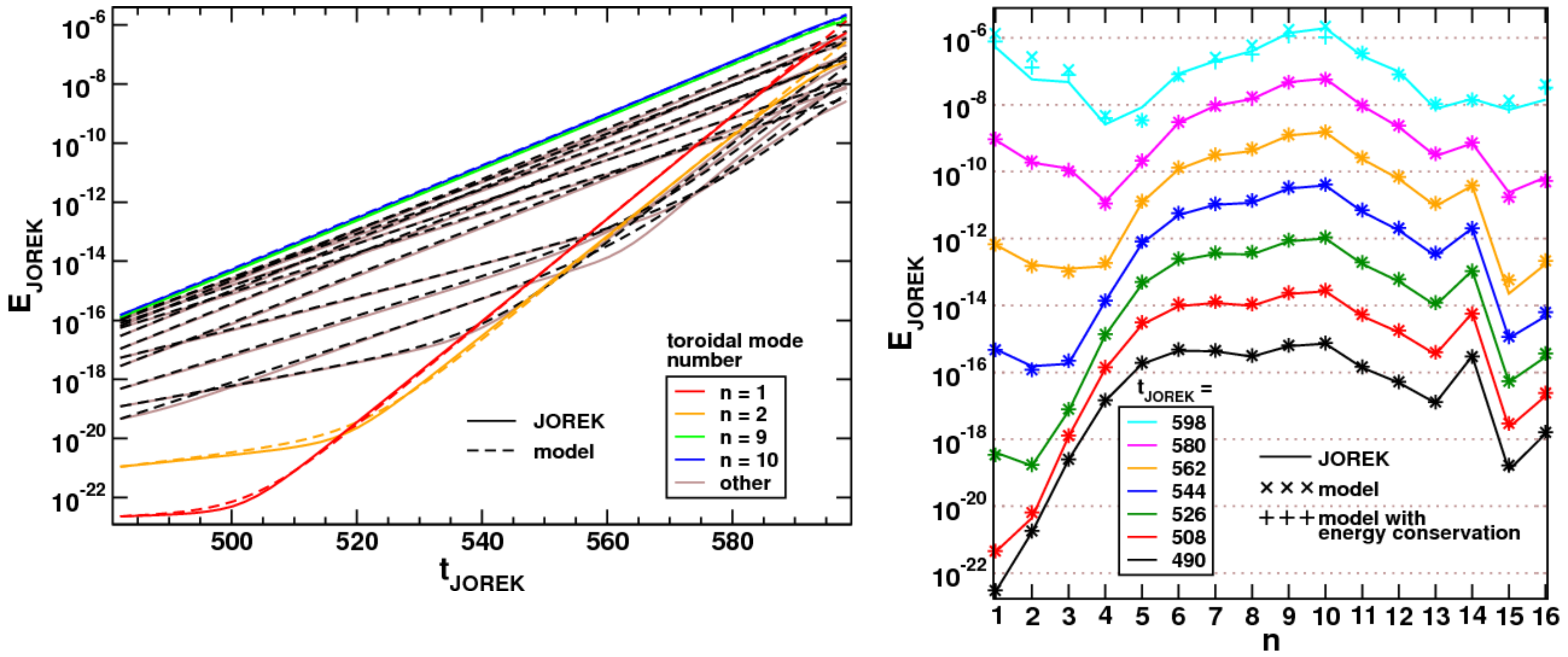
Nonlinear evolution of the toroidal harmonics

linear phase
- constant growth rates
- harmonics grow independently
- intermediate mode numbers are most unstable

early nonlinear phase
- growth rates influenced by interaction between harmonics
- low-n harmonics become important

saturation
- growth rates decrease due to change of background profiles

↪ **experiment**
strong low-n components of magnetic perturbations in ELMy discharges in TCV [1]



Time evolution of the energies of the toroidal Fourier spectrum in a JOREK simulation with n=1, 2, ..., 16 compared to the results of the simple quadratic mode coupling model.

Simple quadratic coupling model

idea
energy transfer among toroidal harmonics via quadratic mode coupling ("sum and difference mode number generation")

- superposition of toroidal harmonics j & k

$\xrightarrow{\text{quadratic terms}}$ |j±k| components are generated

⇒ set of coupled nonlinear differential equations:
for i = 1, ..., 16

$$\frac{\partial A_i}{\partial t} = \gamma_i A_i + \sum_{j=1}^{16} \sum_{k=1}^{16} \gamma^i_{jk} A_j A_k \delta(i \pm j \pm k)$$

where A_i = amplitude of i^th toroidal harmonic
γ_i = linear growth rates
γ^i_{jk} = coupling constants

- assumes mode rigidity (constant γ^i_{jk})
- no saturation effects included (constant γ_i)

- only very few of the nonlinear terms determine evolution

⟹ **reproduces and explains JOREK results** in early nonlinear phase

References

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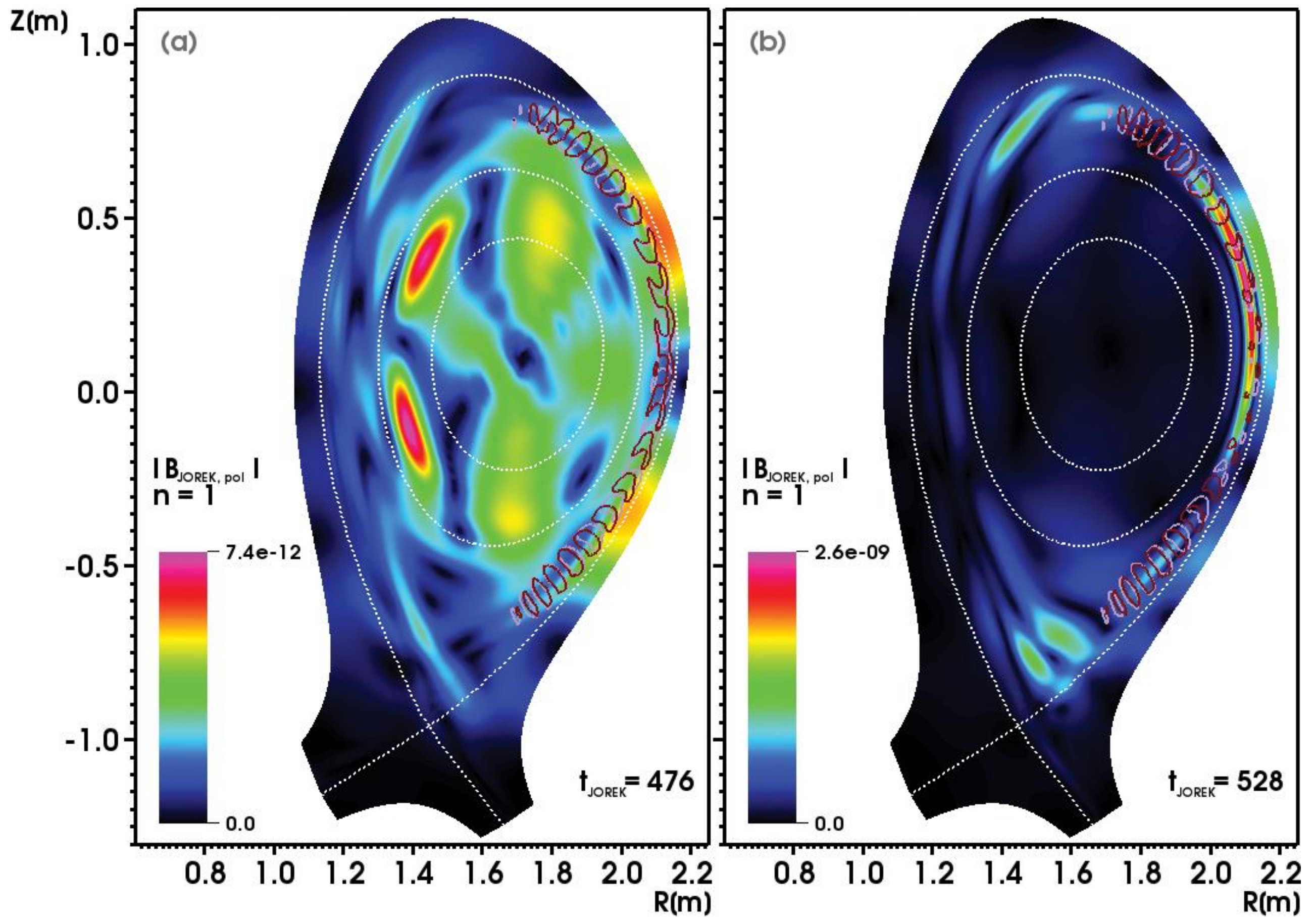

Poloidal cross section of the absolute value of the n=1 poloidal magnetic field perturbation in the linear phase (a) and in the early nonlinear phase (b). The dotted white lines show flux surfaces at ψ_N=0.33, 0.66 & 1.0 and contours at 50% of the maximal absolute value of the poloidal magnetic field perturbation are shown for the n=9 (mauve) and the n=10 component (dark red).

Evolution of the n=1 spatial structure

spatial structure of n=1 changes due to nonlinear drive

- linearly unstable n=1: broad structure in core region (slow rigid growth)

⇓ transition

- nonlinearly driven n=1: localized at edge (fast rigid growth)

↳ localization of driven n=1:
  - radially: where dominant driving harmonics are localized
  - poloidally: where dominant driving harmonics are in phase

Conclusions

- linearly weakly unstable toroidal harmonics can achieve large growth rates driven by nonlinear coupling
- in particular, the n=1 nonlinearly becomes comparable to linearly dominant harmonics
- strong low-n components have also been observed in experiments
- simple quadratic interaction model reproduces and explains the evolution of the toroidal Fourier spectrum
- spatial structure of the n=1 changes from broadly expanded over the plasma core in the linear phase to peaked at the plasma edge when nonlinearly driven