A self-sustaining mechanism that prevents tokamak plasmas from sawtoothing in non-linear 3D MHD simulations

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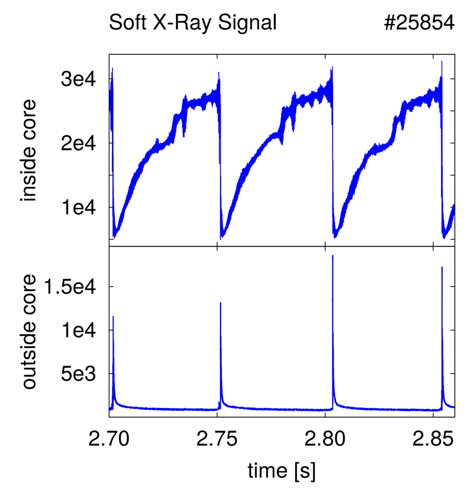
Sawtooth instability

- core relaxation-oscillation instability in tokamak plasmas
- occurs if $q_0 < 1$

- sawtooth cycle:
  - core temperature and density increase slowly
  - (1,1) helical magnetic perturbation arises
  - core pressure is suddenly expelled

- can be detrimental through NTM seeding



Hybrid discharges

- sawtooth-free discharges, e.g. [1-2]
- generated by additional heating during current-ramp phase

- central q-profile flat & slightly above 1
- transport simulations predict q_0 to drop below 1
- current is redistributed by unknown mechanism ("flux pumping")

- relevant for "advanced tokamak" scenarios

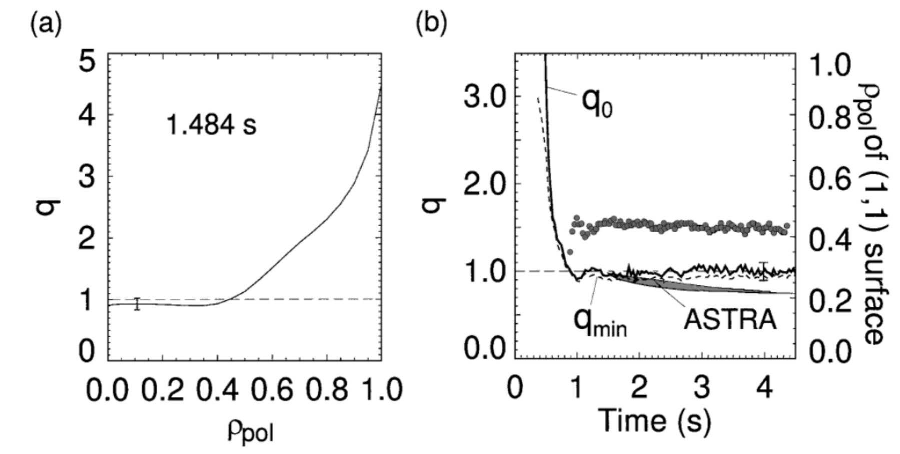


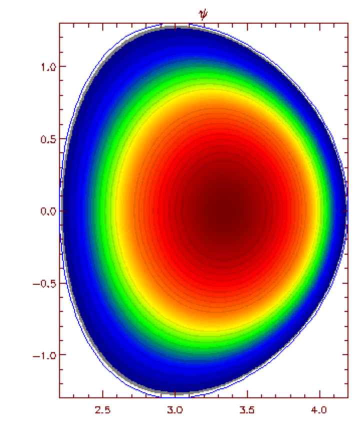
Figure taken from Ref. [1]

M3D-C1 Code

- non-linear 3D two-fluid MHD code
- developed by S.C. Jardin & N. Ferraro [3]

- high-order finite elements:
  - poloidal plane: reduced quintic
  - toroidal direction: hermite cubic
- fully implicit time stepping

- highly versatile, options for:
  - linear, 2D & 3D non-linear
  - straight cylinder & toroidal geometry
  - various MHD models, from reduced resistive to full two-fluid MHD



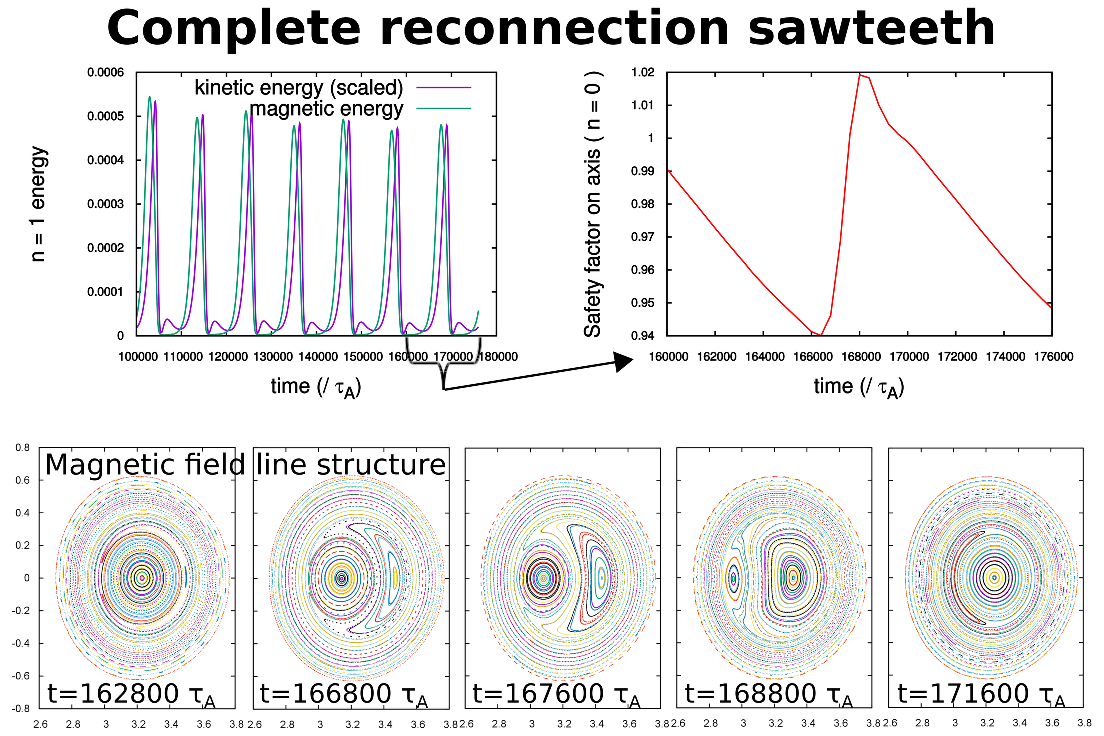
Simulations

- single-fluid full MHD model
- 3D non-linear (& 2D non-linear for comparison)
- toroidal geometry, fixed boundary, 8 tor. planes

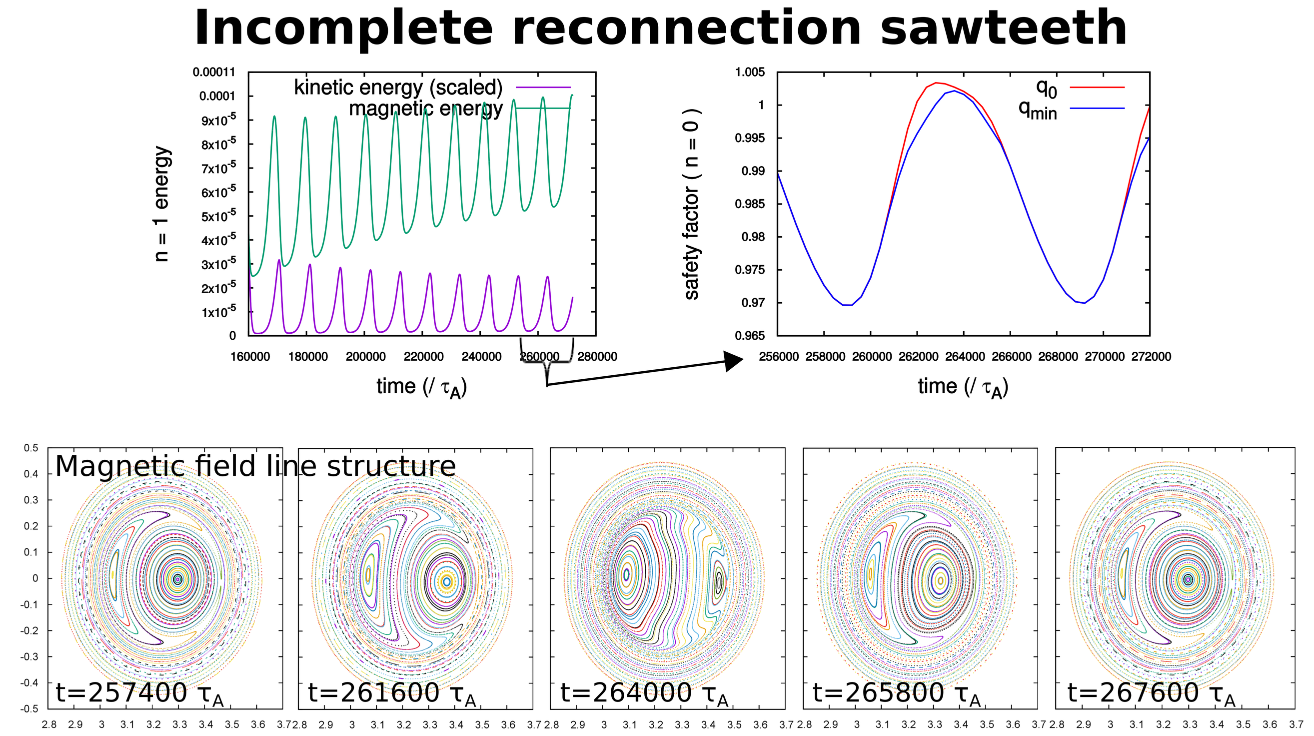
- focus on long-term behavior
  - determined by sources & diffusion coefficients
- varied parameters:
  β, $\kappa_\perp$ & heat source S_T, shape of S_T

- Spitzer resistivity scaled to be similar for all runs
- comparison with experimental parameters:
  $\eta \approx 4\cdot 10^{-6}\,\Omega\mathrm{m} \approx 10^3 \cdot \eta_{exp}$
  $\kappa_\perp/\eta \approx \kappa_{\perp,exp}/\eta_{exp}$

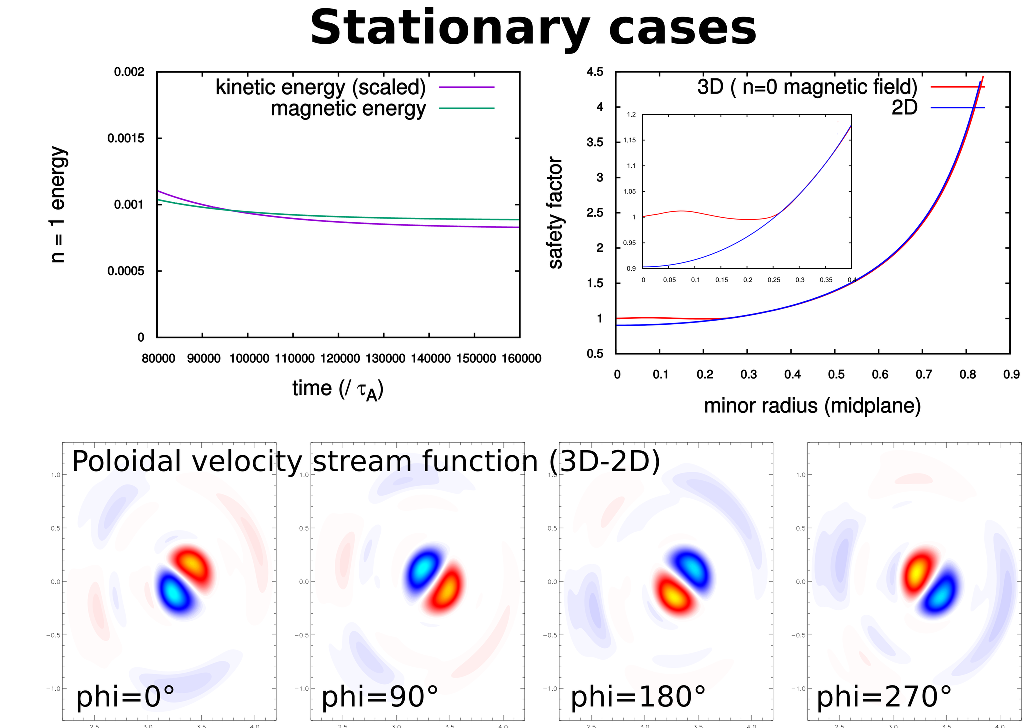
Complete reconnection sawteeth



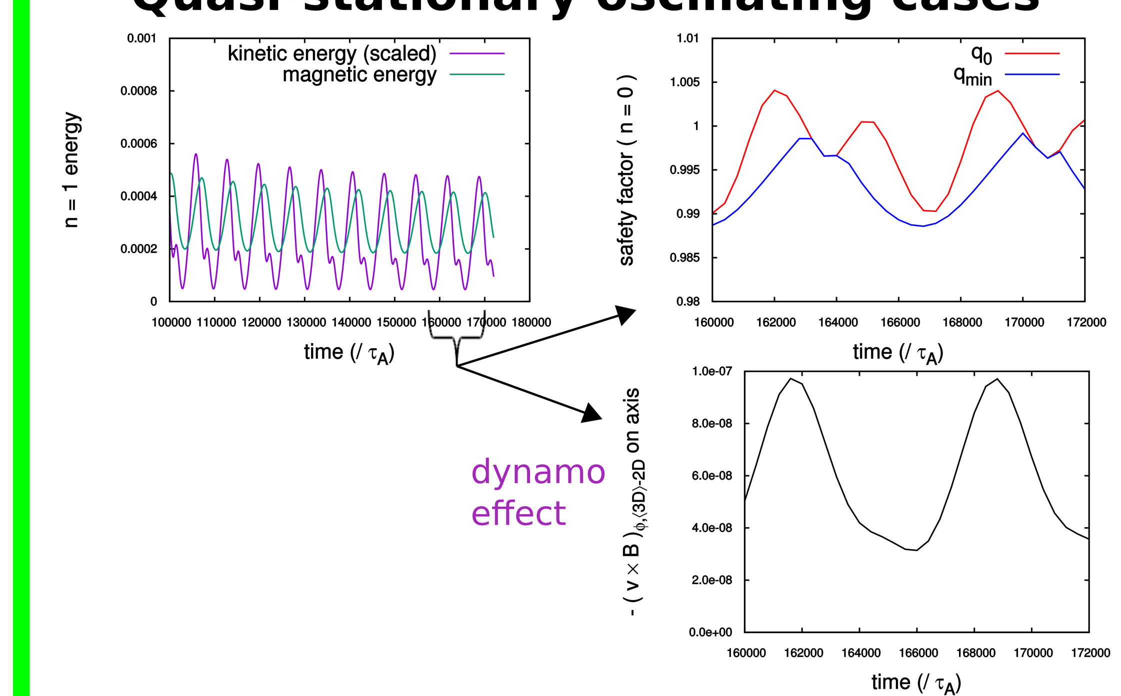
Incomplete reconnection sawteeth



Stationary cases



Quasi-stationary oscillating cases



Current flattening mechanisms [4]

$$\partial_t \mathbf{B} = -\nabla\times\mathbf{E} \quad \text{and} \quad \mathbf{B} = \nabla\times\mathbf{A} \quad\Rightarrow\quad \partial_t\mathbf{A} = -\mathbf{E} - \nabla\Phi + \frac{V_L}{2\pi}\nabla\phi$$

insert $\mathbf{A} = R^2\nabla\phi\times\nabla f + \Psi\nabla\phi - F_0\ln R\hat{Z}$ and $\mathbf{E} = \eta\mathbf{J} - \mathbf{v}\times\mathbf{B}$
take toroidal component $R^2\nabla\phi\cdot[\ ]$

$$\Rightarrow \partial_t\Psi = -R\eta J_\phi + R\hat{\phi}\cdot(\mathbf{v}\times\mathbf{B}) - R\hat{\phi}\cdot\nabla\Phi + \frac{V_L}{2\pi}$$

write in terms of $n=0$ and $n=1$ components and take toroidal average
$(\mathbf{v}_0 = 0,\ \nabla\Phi_0 = 0)$

$$\Rightarrow \partial_t\Psi_0 = -R\eta_0 J_{\phi,0} - R\eta_1 J_{\phi,1} + R\hat{\phi}\cdot(\mathbf{v}_1\times\mathbf{B}_1) + \frac{V_L}{2\pi} \qquad \langle 3D\rangle$$

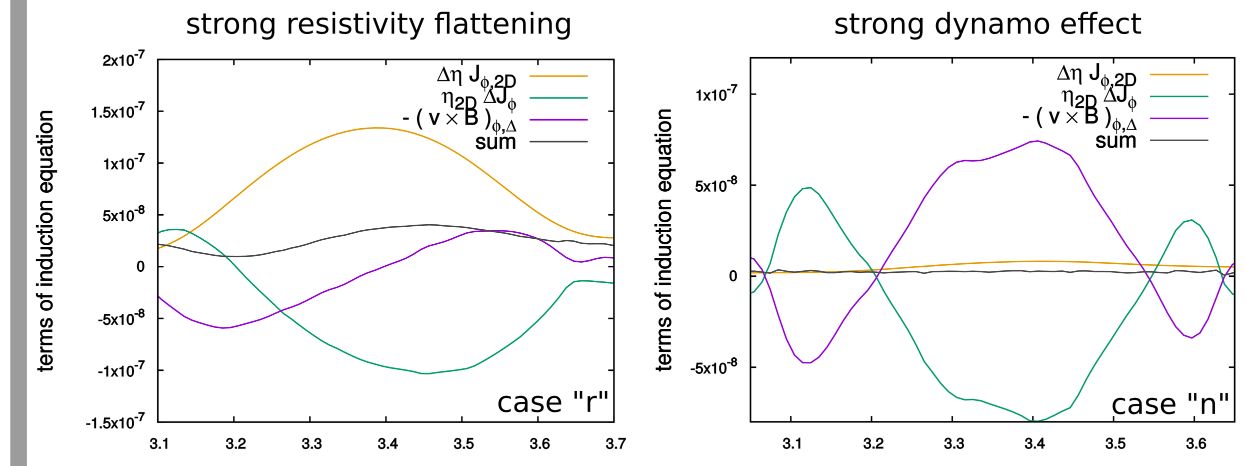
linearize the $\langle 3D\rangle$ equation around the 2D solution:

$$0 = -R\eta_{2D}J_{\phi,2D} + \frac{V_L}{2\pi}$$

define $\Delta J_\phi = J_{\phi,0} - J_{\phi,2D}$ and $\Delta\eta = \eta_0 - \eta_{2D}$

$$\Rightarrow \partial_t\Psi_0 = -R\Delta\eta J_{\phi,2D} - R\eta_{2D}\Delta J_\phi + R\hat{\phi}\cdot(\mathbf{v}_1\times\mathbf{B}_1) - R\eta_1 J_{\phi,1}$$

vanishes for stationary cases & for quasi-stationary cases in time-average — resistivity flattening — resulting current flattening — dynamo effect — small



Varied parameters

$$\beta = \frac{2\mu_0}{B_\theta^2}\int_0^{r_1}\left(\frac{r}{r_1}\right)^2\left(-\frac{dp}{dr}\right)dr$$
→ linear drive of instability that enables current flattening

perpendicular heat diffusion coefficient $\kappa_\perp$ & heat source S → stiffness of temperature profile

peakedness of heat source S → how strong the current flattening effect needs to be in order to keep q_0 at unity

$$\eta J_{\phi,0}(q_0=1) - \eta J_{\phi,0}(q_0=q_{0,2D}) \approx \frac{2\eta B_{\phi,0}}{\mu_0 R_0}\frac{(1-q_{0,2D})}{q_{0,2D}}$$

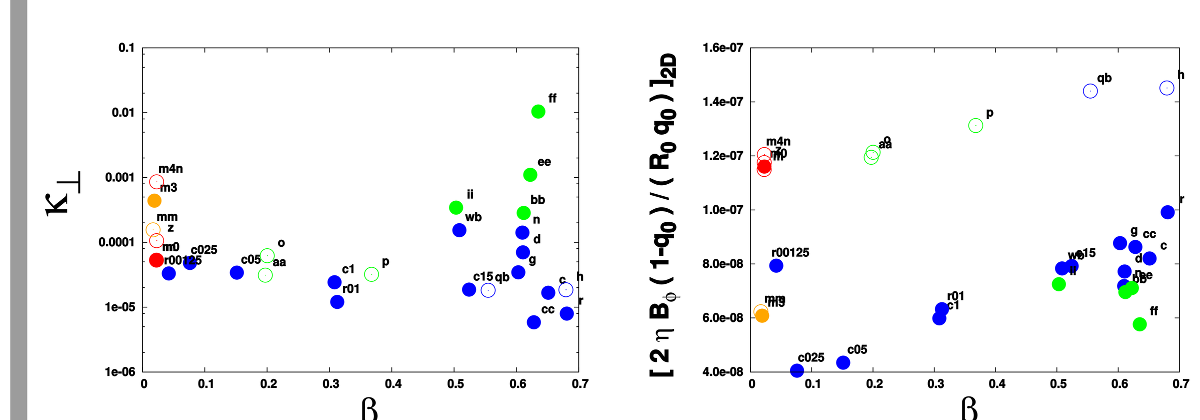
"Flux-pumping" [4]

central q flat & close to unity → low shear pressure driven instability → (m=1,n=1) helical flow → convective resistivity flattening / dynamo loop voltage → central q flat & close to unity

References
[1] Gruber, O. et al., Phys. Rev. Lett. 83 (1999)
[2] Petty, C. et al., Nucl. Fusion 56 (2015)
[3] S.C. Jardin et al., Computational Science and Discovery 5 (2012)
[4] S.C. Jardin et al., Phys. Rev. Lett. 115 (2015)

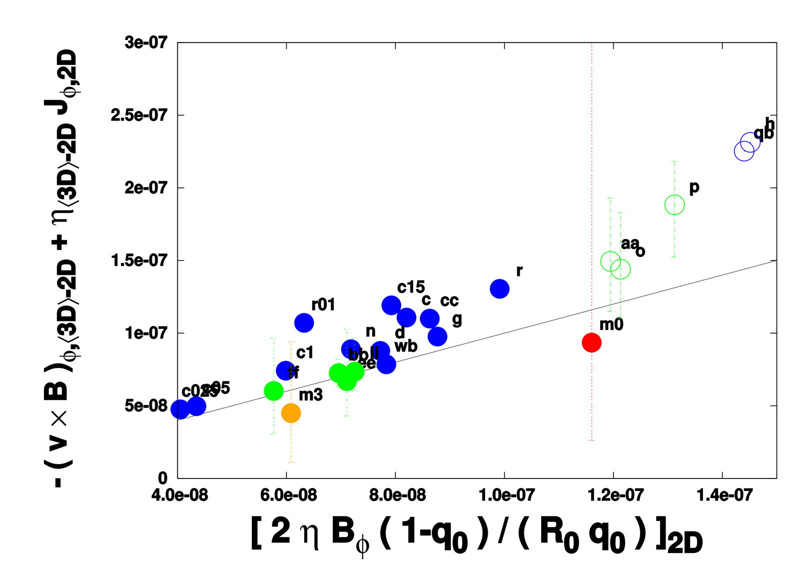
Regime of sawtooth-free states

Sawtooth-free states occur if beta is high enough...



... so that the current flattening mechanisms can keep q_0 close to 1

resistivity flattening + dynamo effect



Complete reconnection sawteeth
Incomplete reconnection sawteeth
Stationary state
Quasi-stationary oscillating state

○ more peaked heat source
● less peaked heat source

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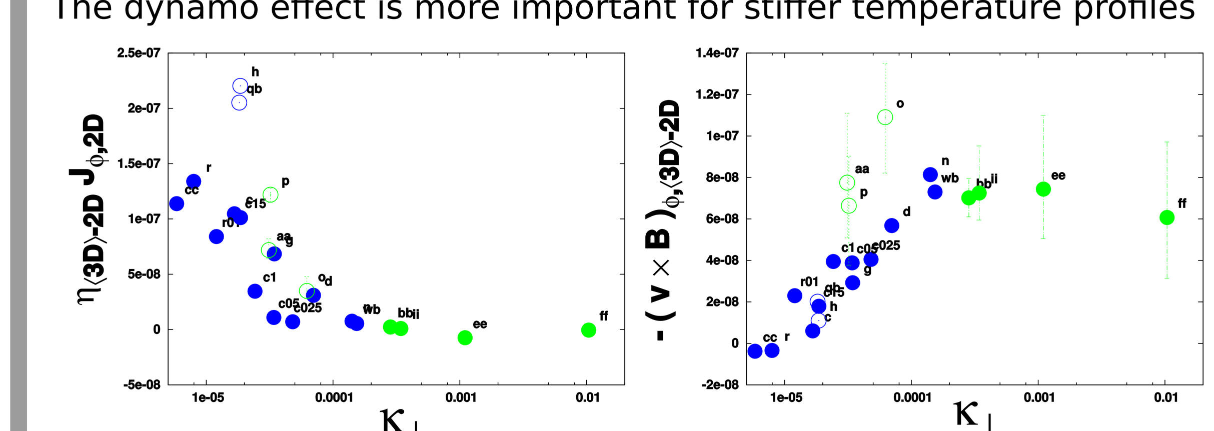
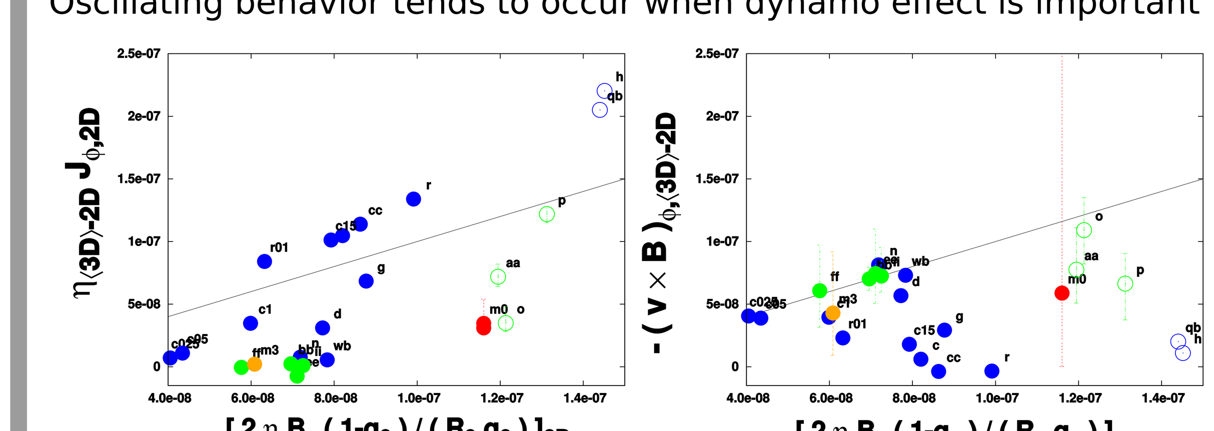
Which mechanism is dominant?

The dynamo effect is more important for stiffer temperature profiles



Oscillating behavior tends to occur when dynamo effect is important



Summary

Types of long-term behavior

- low beta → **Sawtoothing**
  - high Δq_0 → **Complete reconnection**
  - low Δq_0 → **Incomplete Reconnection**
- high beta → **Sawtooth-free**
  - low $\kappa_\perp$ & S_T → **Stationary**
  - high $\kappa_\perp$ & S_T → **Oscillating**