Implementation of a fluid model for the non-linear interaction between runaway electrons and background plasma

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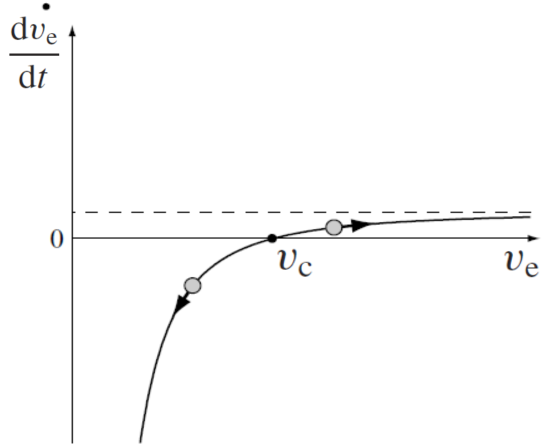
Runaway electrons – basic overview

- Rapid fall of collision frequency at high energies, $\nu_{\mathrm{ei}} \sim \nu_{\mathrm{ee}} \sim 1/v_{\mathrm{e}}^3$

  ⟹ Free acceleration or "runaway" of electrons



- $v_c \approx 6 v_{Te}$ in a normal Tokamak discharge
  - Runaway of only distribution tails

- Further, $\frac{v_{\mathrm{c}}^2}{v_{T\mathrm{e}}^2} = 78 \frac{n_{20}}{|E_{\|}| T_{\mathrm{k}}}$
  - At $E > E_c$, runaway of thermal electron population*,
  - Typical $E/E_c \sim 10^{-2}$

$$E_c = \frac{n_e e^3 \ln \Lambda}{4\pi\epsilon_0^2 m_e c^2},$$

Larger E-fields necessary for significant runaway electron generation

*Dreicer, 1959

Scenario in a disruption

Field stochastization

↓

Large radial transport

↓

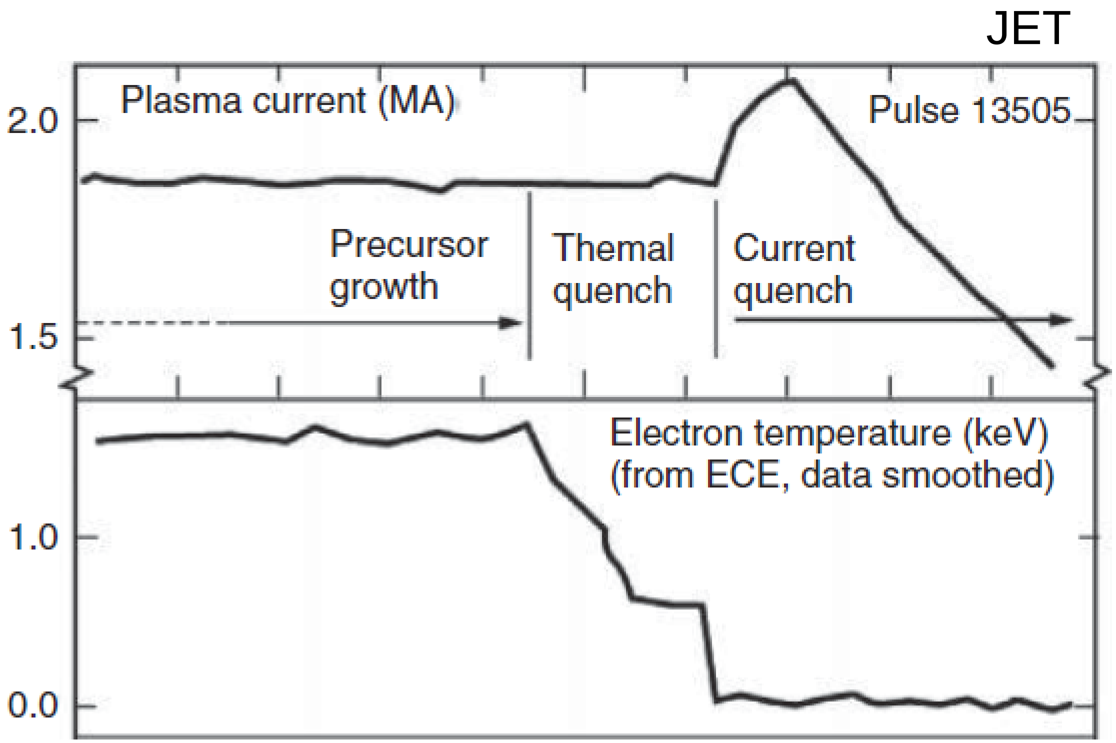
Thermal quench

↓

Large toroidal E

↓

RE generation



Wesson et al. 1989

Implications for large tokamaks (like ITER)

- Up to 70% conversion to RE current
- If unconfined => potentially severe localized surface damage*

**Hollmann PoP 2015*

Runaway confinement and MHD

- RE confinement depends on flux-surface restoration timescale
- Plasma stability is strongly affected by REs
- RE ⬄ MHD is hence important and is also highly non-linear

Larger aim: Understand the coevolution of disruption and runaway electrons using a fluid model

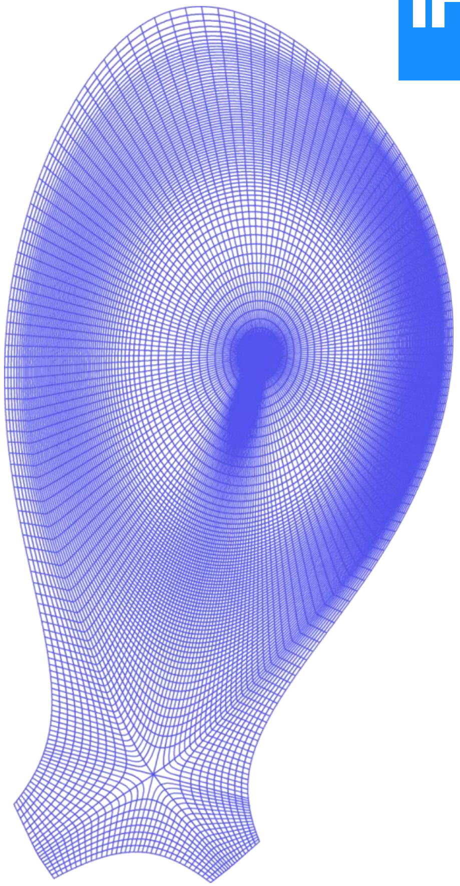
Scope of this talk: A fluid model for REs in the MHD code JOREK, tested with artificial thermal quenches

Non-linear MHD code JOREK*

- Single fluid reduced-MHD code
- Realistic toroidal X-point geometries
- Includes 3D resistive wall effects

Numerics

- Flux-aligned 2D Bezier finite-elements
- Fourier decomposition in the toroidal direction
- Full implicit time-stepping
- Preconditoning + GMRES iterations
- MPI + OpenMP parallelized



Flux-aligned 2D Bezier finite elements

Routinely used to simulate ELMs and disruptions

Huijsmans et al., NF 2007

RE fluid model

REs considered as a separate fluid species

Total current density: $\boldsymbol{j} = ne\left(\boldsymbol{v}_i - \boldsymbol{v}_e\right) + \boldsymbol{j}_r$

Runaway current density: $\boldsymbol{j}_r = -en_r\boldsymbol{v}_r$

Integrating the drift-kinetic equation over the velocity phase-space yields

$$\frac{\partial n_r}{\partial t} + \boldsymbol{v}_r.\nabla n_r = S_p + S_s \qquad \boldsymbol{v}_r = \frac{c\boldsymbol{B}}{B} + \frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2}$$

Dreicer source (small angle Coulomb scattering)

$$S_p^* = \left(0.21 + 0.11Z_e\right) n_e\nu_{ee}\epsilon_d{}^{-\frac{3}{16}(1+Z_e)} \exp\left(-\frac{1}{4}\epsilon_d{}^{-1} - (1+Z_e)^{\frac{1}{2}}\epsilon_d{}^{-\frac{1}{2}}\right)$$

$$\exp\left[-\frac{T_e}{m_e c^2}\left(\frac{1}{8}\epsilon_d{}^{-2} + \frac{2}{3}(1+Z_e)^{1/2}\epsilon_d{}^{-3/2}\right)\right]$$

* *Connor et. al., NF (1975)*

RE fluid model

Avalanche growth (large angle knock-on collisions)

$$S_s^* = n_r \nu_{fp} \frac{\epsilon_c - 1}{\ln \Lambda} \sqrt{\frac{\pi \varphi}{3\left(Z_e + 5\right)}} \left(1 - \frac{1}{\epsilon_c} + \frac{4\pi \left(Z_e + 1\right)^2}{3\varphi \left(Z_e + 5\right)\left({\epsilon_c}^2 + 4/\varphi^2 - 1\right)}\right)^{-1/2}$$

Other governing equations (full form)

$$\partial_t \rho = -\nabla \cdot (\rho \boldsymbol{v}) + \nabla \cdot (D \nabla \rho)\,,$$

$$\rho \partial_t \boldsymbol{v} = e n_r \boldsymbol{E} + (\boldsymbol{j} - \boldsymbol{j}_r) \times \boldsymbol{B} - \rho \left(\boldsymbol{v} \cdot \nabla\right) \boldsymbol{v} - \nabla p - \boldsymbol{v} S_\rho - \nabla \cdot \overline{\overline{\Pi}} - \boldsymbol{S}_\nu,$$

$$\partial_t p = -\left(\boldsymbol{v} \cdot \nabla\right) p - p \nabla \cdot \boldsymbol{v} + (\gamma - 1)\, \nabla \cdot (-k \nabla T) + (\gamma - 1)\, \eta |\boldsymbol{j} - \boldsymbol{j}_r|^2,$$

$$\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B} + \eta\, (\boldsymbol{j} - \boldsymbol{j}_r) + \frac{(\boldsymbol{j} - \boldsymbol{j}_r) \times \boldsymbol{B} - \nabla p_e}{n e},$$

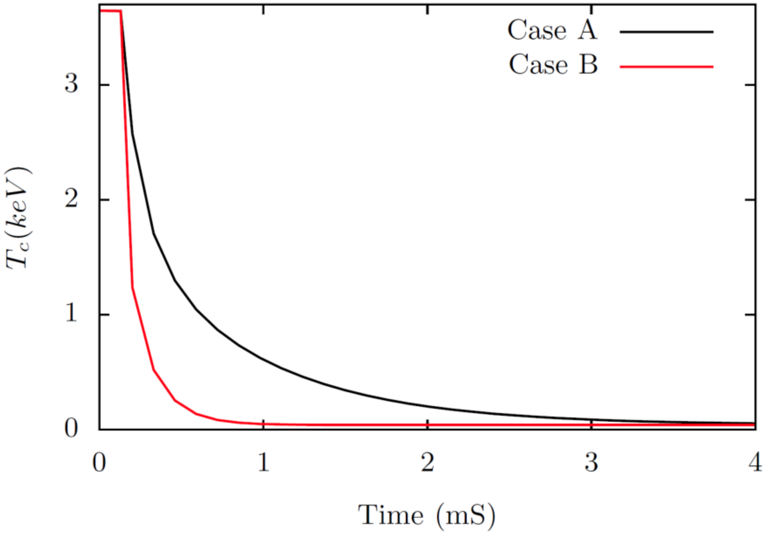
$$\partial_t \boldsymbol{B} = -\nabla \times \boldsymbol{E}.$$

* *Rosenbluth et al., NF (1997)*

Test cases with pseudo thermal quenches

Equilibrium plasma quenched by sudden step-up of perp. thermal conductivity

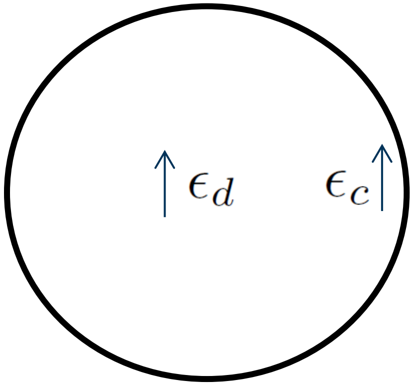
	Case A	Case B
$k_\perp (normalized)$	0.5×10^{-4}	2.5×10^{-4}
Pre-quench ϵ_D (Centre)	0.67%	0.67%
Pre-quench ϵ_c (Centre)	1.8	1.8



RE source triggering thresholds

$$S_p = 0, \epsilon_d < 1\% \text{ *}$$

$$S_s = 0, \epsilon_c < 1.7$$



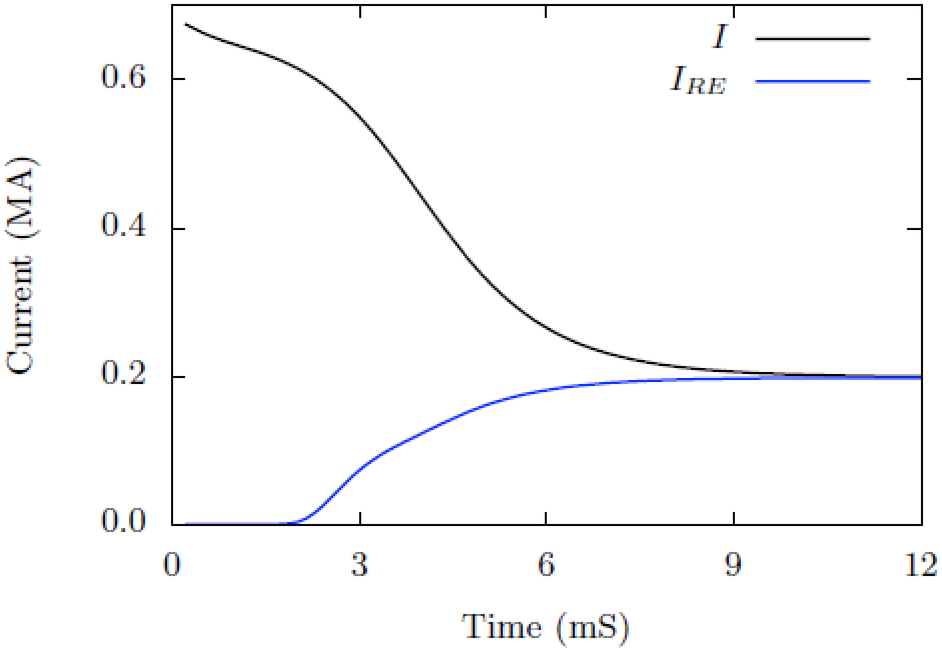
Circular plasma

$R = 1.65m, a = 0.6m,$
$\mathrm{I} = 0.67\mathrm{MA}$
(sim. to ASDEX-U)

* *Stahl et. al., PRL (2015)*

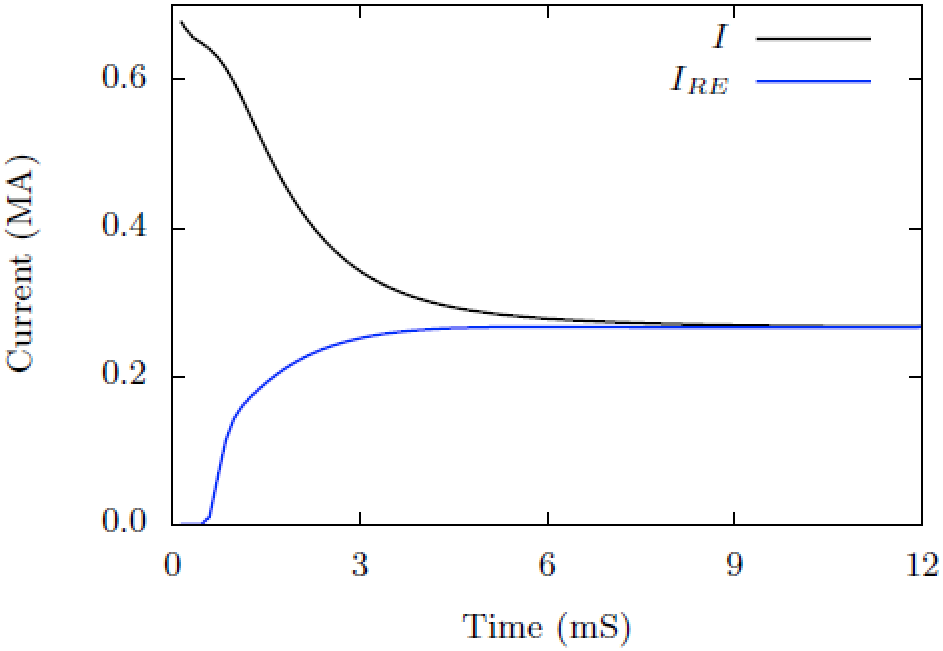
Runaway conversion

Case A



RE Conversion = 29.4%

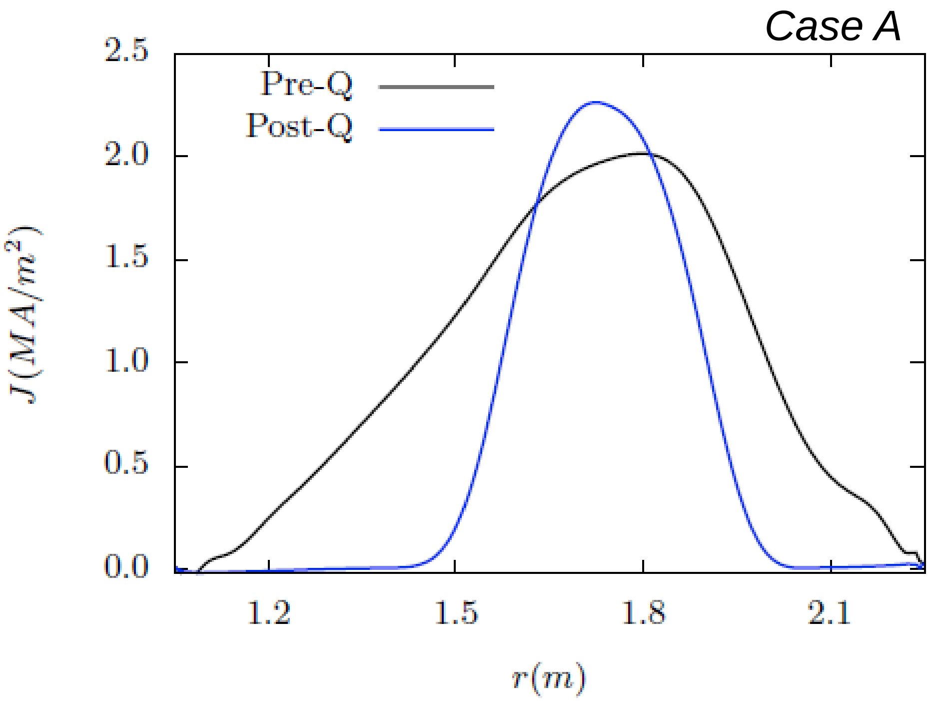
Case B

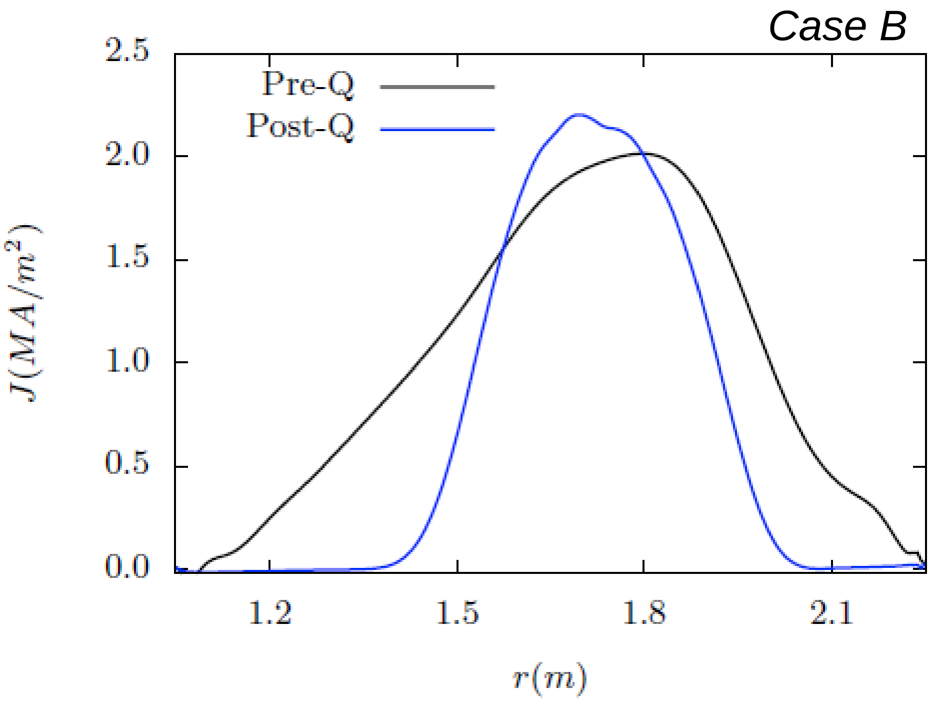


RE Conversion = 39.5%

Qualitatively similar behaviour observed experimentally

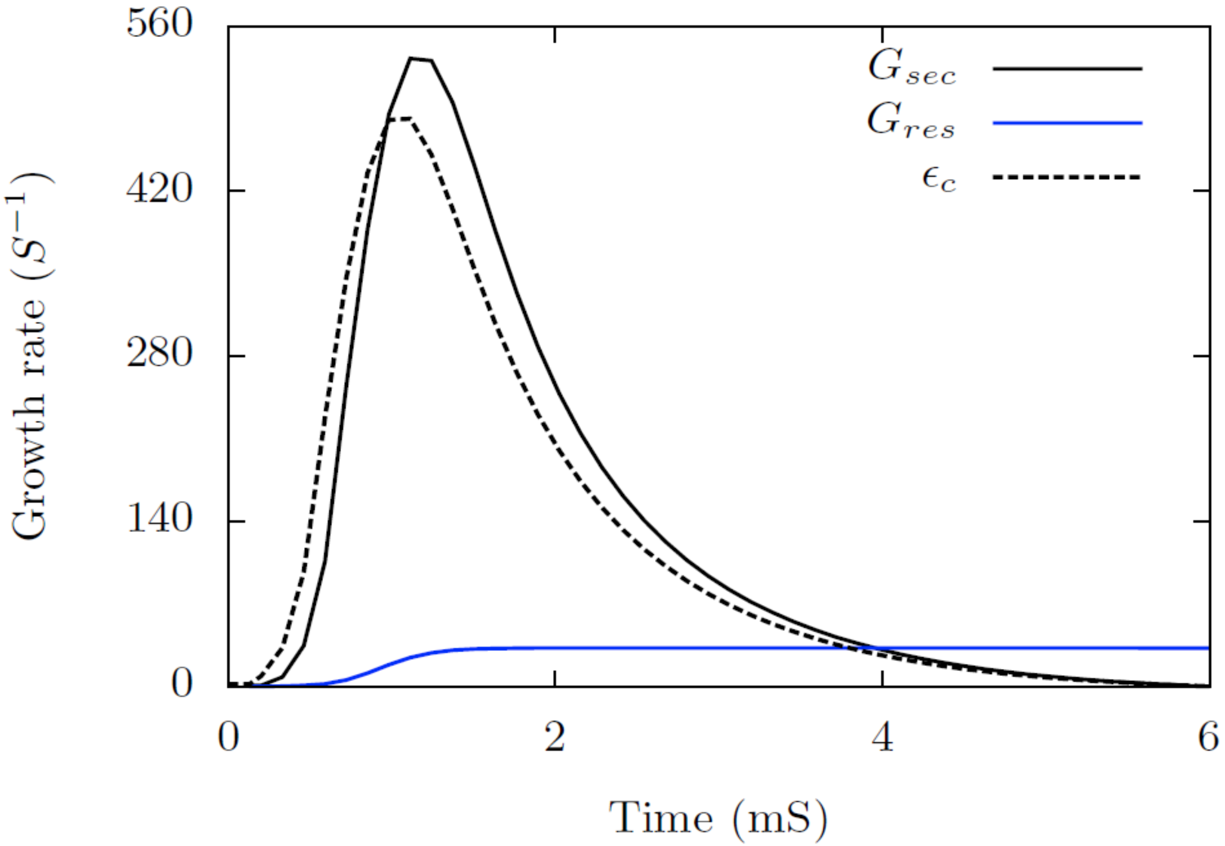
Peaking of RE profiles





Reduced peaking for larger conversions

Growth rates

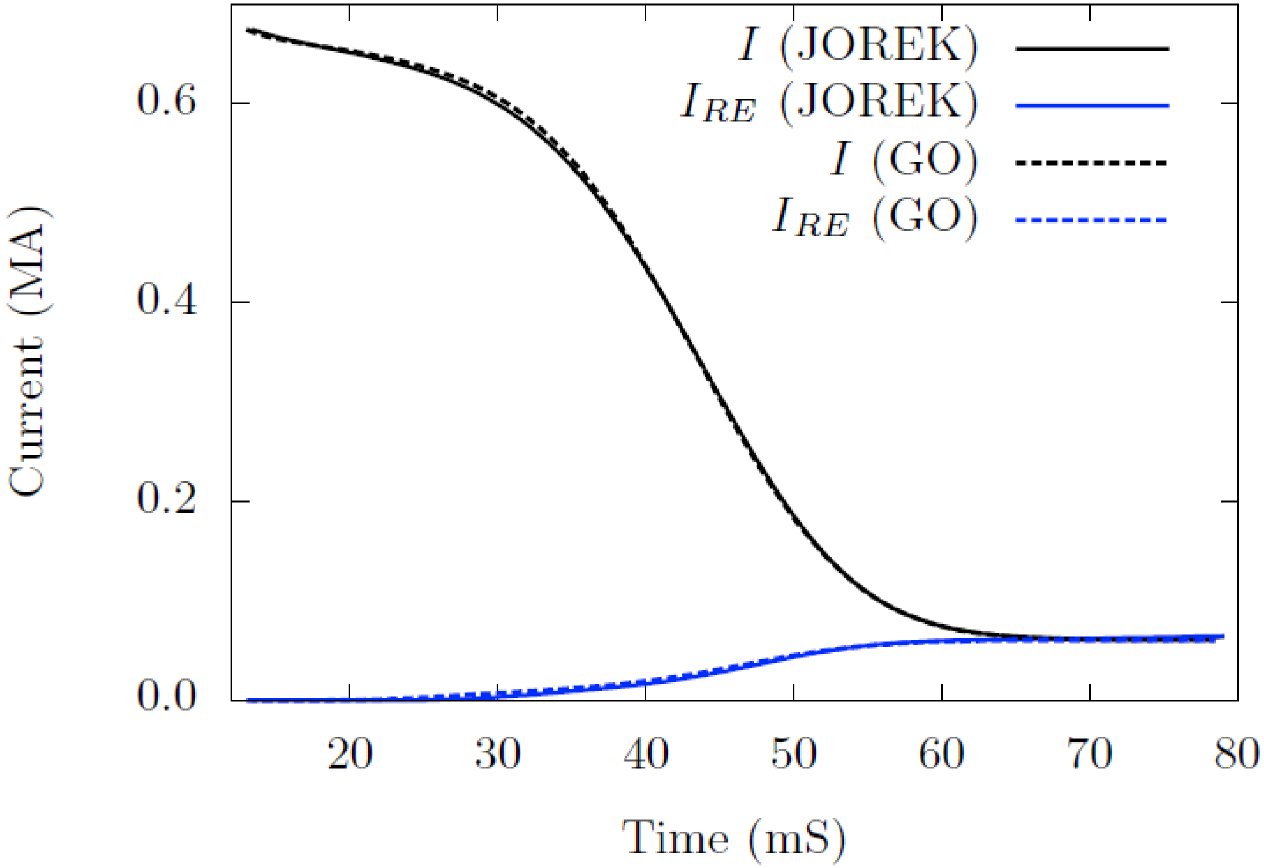

Case B

$$G_{sec} = \frac{S_s}{n_r}$$

$$G_{res} = \frac{\eta}{\mu_0 L^2}$$

RE current growth saturates when E-field diffusion dominates

Preliminary comparison with GO*



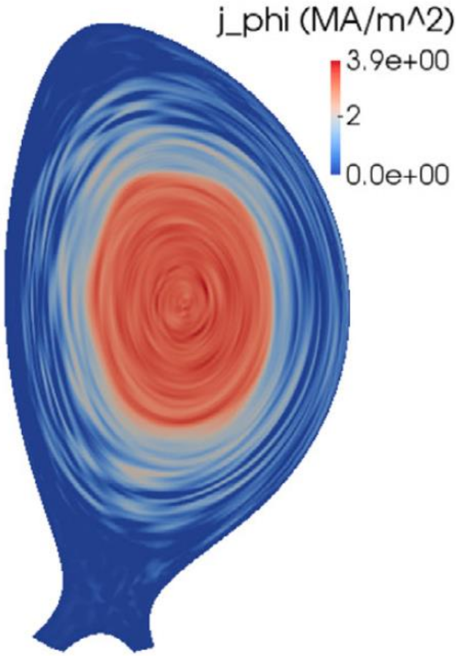
Much slower quench

** GO: Existing 1D RE code*

** Pokol et al., EPS conf. (2017)*

Outlook

- Improved near-threshold treatment*
- Investigate the non-linear interaction of resistive-kink modes with peaked RE beams
- Simulate the interaction of REs with a real disrupted plasma
    - Detailed comparison to ASDEX-U and other machines



Nardon et al., PPCF (2017)

* *Embreus et al., REM talk. (2017)*

Backup

References

1. *P. Helander, D. Grasso, R.J. Hastie, A. Perona, Resistive stability of a plasma with runaway electrons, Phys. Plasmas* ***14****, 122102 (2007).*
2. *J.W. Connor, R.J. Hastie, Relativistic limitations on runaway electrons, Nucl. Fusion 15, 415 (1975).*
3. *M.N. Rosenbluth, S.-V. Putvinski, Theory for avalanche of runaway electrons in Tokamaks, Nucl, Fusion 37, 1355 (1997).*
4. *P. Helander, L.-G. Eriksson, F. Andersson, Suppression of runways electron avalanches by radial diffusion, Phys. Plasmas 7, 4106 (2000).*
5. *P. Aleynikov, B.N. Breizman, The theory of two threshold fields for relativistic runaway electrons, Phys. Rev. Lett. 114, 155001 (2016).*